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Buying Shares and/or Votes for Corporate Control

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We explore how allowing votes to be traded separately of shares may affect the efficiency of corporate control contests. Our basic set-up and the nature of the questions continue the work of Grossman and Hart (1980), Harris and Raviv (1988), and Blair, Golbe and Gerard (1989). We consider three cases with respect to the allowable price offers (for shares and for votes when they can be traded separately): unrestricted price offers, quantity-restricted price offers, and price offers contingent on winning. Our main results are characterizations of the equilibria and of the circumstances under which vote buying is harmful. We show that allowing votes to be traded separately of shares results in inefficiencies in all the cases we study. Similarly allowing quantity-restricted offers is also harmful, but allowing conditional offers is not in itself detrimental to efficiency. The paper also makes a methodological contribution to the analysis of takeover games with atomless shareholders. It provides a way of dealing with asymmetric equilibria that must be dealt with for a complete analysis and it proves existence of an equilibrium.

Key words: Vote-buying, Corporate control contest

JEL Codes: C72, D72, G34

1. INTRODUCTION

We study contests over the control of a firm with widely dispersed ownership. The focus is on the implications of allowing the sale of votes separately from shares. There is a substantial recent literature arguing that vote buying occurs in practice (albeit indirectly) but we are unaware of any model that fully characterizes and contrasts the equilibrium outcomes with and without vote trading and that pinpoints the effect on efficiency and shareholder profits of allowing for separate vote buying.

This paper is a direct follow-up on the early literature on the allocation of voting rights to shares which goes back to Grossman and Hart (1980), Harris and Raviv (1988) and Blair, Golbe and Gerard (1989). While our basic set-up and the nature of the questions follow this literature, the results obtained are new. A more detailed discussion of the relation to the literature is presented in Section 2 below.

Following the literature, our model features two contestants competing for control—an incumbent and a rival. The rival moves first and makes a tender offer to the shareholders. The incumbent responds with a competing offer. Then the shareholders simultaneously make their tendering decisions that determine which contestant obtains control. The firm generates income for its shareholders and a private benefit for the party in control; the magnitudes of the income and benefits depend on the identity of the parties. In addition to outright offers for shares (and for votes when such offers are permitted), we allow the contestants to make either conditional offers (contingent on winning) or restricted offers (placing a cap on the quantity of shares that will be purchased at the announced price).[1] We show that allowing vote buying is (always weakly and sometimes strictly) harmful in terms of efficiency in all versions of the model (*i.e.* whether or not quantity restrictions are allowed and whether or not conditional offers are allowed). Allowing restricted offers is also harmful to efficiency (whether or not vote buying is allowed). However, allowing conditional offers is not in itself detrimental to efficiency. There are of course other considerations like the presence of taxation (Blair, Golbe and Gerard, 1989) under which vote buying might increase efficiency. The present work highlights the costs directly resulting from the forms of contracts allowed.

A main contribution is an exact characterization of when and why vote buying is harmful, which should enable future work to contrast more precisely the costs and benefits of vote buying. The sharp observations we obtain regarding efficiency no longer hold if we look at the profits of the initial shareholders alone (ignoring the benefits of control). In particular, there are parameters for which allowing separate vote trading increases shareholder profits, despite being harmful for efficiency.

Besides the substantive insights outlined above, the paper also has a methodological contribution to the analysis of takeover games with a continuum of shareholders. It provides a way of dealing with the mixed or asymmetric strategies that are crucial for the analysis.[2]

Indeed, we show that the asymmetric equilibria play two crucial roles in generating the inefficiencies of vote trading. (This is the case despite the fact that we also prove that along the equilibrium path only symmetric strategies are used.) First, an inefficient rival may make a pre-emptive offer against which the only winning counter-offers of the efficient incumbent lead to subgames with asymmetric equilibria that cause the incumbent to incur losses. Second, asymmetric equilibria in off-path subgames can prevent an efficient rival from making a profitable offer because the inefficient incumbent can subsequently lead play to a subgame with an asymmetric equilibrium that results in losses to the rival. Interestingly, the asymmetric equilibria effectively enable the incumbent to obtain the same outcome as results from using a quantity-constrained offer. These intuitions are explained in more detail in Sections 4.2 (before Theorem 2) and 4.2.1.

The paper also develops arguments that facilitate characterization results without fully constructing the set of equilibria and deals with the question of existence. Thus, this contribution provides a full characterization of equilibria that can be used to study these and related issues.

The original motivation for our interest was to understand the difference between the acquisition of control in the corporate context and vote buying in elections in the political context. Intuitive discussions tend to view the former activity as efficiency enhancing and the latter as detrimental and it is interesting to understand whether and in what sense this might be true. This question has already been discussed to some extent by Dekel, Jackson and Wolinsky (2008). The present analysis deepens the understanding by emphasizing that, in the corporate arena, the acquisition of control could be associated with efficiency only because shares are traded with the votes. Vote buying alone is not efficient in the corporate context as well. In the political arena, there is no natural analog to the trading of shares. Such an analogue would require that

1. We assume small shareholders to rule out equilibria where they are pivotal and assume that the competing parties must make identical offers to all shareholders.

2. Asymmetric strategies (or equilibria) mean throughout that different shareholders make different tendering decisions, despite being identical.

each vote-buying party will receive from (or compensate) the voters who tender their votes to that party any future benefit (or loss) that those voters enjoy (or suffer) from the policies implemented by the winning party. Our analysis does imply that when there are such conditional *ex post* transfers, allowing vote buying would be efficiency enhancing.

2. RELATED LITERATURE

A large literature on the efficiency of takeovers follows the work of Grossman and Hart (1980 and 1980; henceforth GH80 and GH88) and Harris and Raviv (1988; henceforth HR).The main message of GH88 and HR is the optimality of one share–one vote for efficiency and the potential benefits of violating it for maximizing shareholder profits. Our results extend this general message to the important case of vote buying (which while closely related is strictly speaking not covered by their framework) and furnish it with firmer foundation by providing a complete equilibrium analysis. In the remainder of this section, we attempt to place our work in the context of the broader literature, but obviously this is not a comprehensive survey.[3]

One of the first formal papers on takeovers, GH80, considers the case of a single bidder (*i.e.* the incumbent cannot counter-offer) with dispersed ownership of the firm and studies the resulting free-rider problem. A subsequent literature has discussed the role of separating cash flows from voting rights in overcoming this free-rider problem. See, *e.g.* At, Burkart and Lee (2011), Burkart and Lee (2010), Burkart, Gromb and Panunzi (1998), Gromb (1992), and Marquez and Yilmaz (2006).[4]

This is quite different from our model that, following GH88 and HR, considers the case where the incumbent can make a counter-offer. While HR consider equilibria that allow (all) shareholders to be pivotal, we adopt the GH perspective of equilibria where shareholders are not pivotal. We think that pivot considerations are relevant in a situation in which a small number of large shareholders are holding indivisible blocks of shares, whereas ignoring them seems more suitable for a situation in which the shares are widely distributed among many small shareholders, and this is the context of interest to us. (In a related context, Dekel, Jackson and Wolinsky (2008, Section V.C), we argued that pivotal equilibria are not robust.)

In this environment, it is accepted that one share–one vote yields efficient takeovers: “In widely held firms, one share–one vote is optimal only when several bidders compete, as it ensures that the most efficient bidder gains control” (Burkart and Lee, 2008). Our initial result is a small contribution to this commonly held conclusion by making precise a game and its equilibria (and the refinements needed) to obtain such a result when shareholders are not pivotal.

Much of the literature focuses on the effect of dual-class shares and does not explicitly include the case of trading votes separately from shares which we study. That trading votes may be inefficient in environments such as those considered by GH88 is intuitive from arguments regarding the inefficiencies of dual-class shares. But we are not familiar with any model that explicitly demonstrates and identifies the inefficiencies that result from vote buying in such environments, which is the focus of our analysis.

There is also a notable literature on vote trading and, more generally, empty votes (which are different ways of decoupling shares from votes, including direct vote trades—as we consider, using derivatives, and other methods). Hu and Black (2007) discuss the many ways that empty voting can and does occur. They also document cases where it appears to have been harmful.

3. A broader discussion of the literature is contained, for instance, in the recent survey by Burkart, Gromb and Panunzi (1998).

4. Bebchuk and Hart (2001) argue that combining a tender offer for shares and a proxy vote also yields efficiency.

Christoffersen *et al.* (2007) also find evidence of vote trading (specifically in the equity loan market). But they also find that the average vote trades for a zero price, which they argue follows from asymmetric information and facilitates information aggregation. Schouten (2010) discusses further the possibility that vote buying has benefits due to asymmetric information. By contrast with Christofferson *et al.*, Aggarwal, Saffi and Sturgess (2011) and Kalay, Karakas and Pant (2011) find an increase in the cost of a vote near voting events.[5]

We now turn to the theoretical work on efficiency and vote trading *per se*. Blair, Golbe and Gerard (1989) study efficiency and use a basic model that is similar to ours, but they reach the very different conclusion that with *only* contingent offers vote trading does not harm efficiency. Our analysis shows that this result does *not* hold in the natural environment where contenders can make non-contingent offers as well. Based on their result that vote trading is efficient in the basic model, Blair, Golbe and Gerard (1989) go on to argue that in the presence of other elements like taxation, it might be superior to allow vote trading. Of course, if one allows for non-contingent offers as we do, then a trade-off will arise. The complete analysis of the inefficiencies of vote trading that we provide is a necessary first step towards fully comparing such costs and benefits.

Hu and Black (2007) also argue that decoupling votes from shares can be beneficial as it may "strengthen shareholder oversight or, under some circumstances, foster efficient investment decisions," but they note that it may be harmful as well since it can "facilitate insider entrenchment, destabilize dispersed ownership, and, in the case of vote holders with a negative economic interest, sever the usual assumption that shareholders have a common interest in increasing firm value". Our model shows precisely when a form of insider entrenchment is facilitated—in the sense of showing exactly when an inefficient incumbent retains control. Moreover, we also study the additional harmful effect that insiders can be weakened to the point that an inefficient rival can gain control. The analysis of Hu and Black is done without the constraints of a formal equilibrium model and raises interesting questions that seem worth pursuing formally. While it lies outside the scope of the current paper, once again our formal model may facilitate such developments and should be useful for studying the exact trade-offs between the benefits and harms of vote trading.

Kalay and Pant (2009) allow shareholders to buy and sell votes and shares separately by trading derivatives. Thus, they show that one share–one vote is not enforceable in the presence of derivatives. They then argue that shareholders will trade so that the equilibrium will be efficient and shareholders extract the full surplus from the winning bid.[6] Thus, both efficiency and shareholder optimality are obtained. However, their model differs in some crucial respects from ours. First, they do allow for shareholders to be pivotal, which as we argued seems inappropriate for some contexts of interest. Second, while they allow shareholders to trade derivatives to change their holdings from a one share–one vote starting point, they do not allow shareholders to separate and sell their votes directly to the contestants, which is what we study. If that was possible as well, it is not clear what the result would be: it seems possible that the contestants could use offers for separate votes to their benefit and change the efficiency and shareholder revenue results. (Kalay and Pant do consider the case where the rival can trade in the derivatives market, but only in the case where there is a block shareholder.)

5. The papers use different methods to assess these costs.

6. We do not understand their proof since the timing of the game is not clear to us. (The proof of their Lemma II.2. seems to allow in one case the incumbent and in another case the raider to move first.) For some parameter values, their result still seems to us valid, but it is not clear to us whether the strong efficiency and surplus extraction results that build on this lemma hold in general.

There are other papers that study corporate vote buying but not in a takeover context. For example, Brav and Mathews (2011) study how a trader can use derivatives to deviate from one share–one vote. This can be beneficial or harmful, but they show that it is likely to be harmful when shareholders vote correctly and separating votes from shares is inexpensive. This complements our result as their inefficiency is due to another source—a trader short-selling the stocks and then using votes to lower the firm's value. In another vein, Neeman and Orosel (2006) consider a repeated game in which vote buying signals competence and show that if the difference between the value of control and the outside option is increasing/decreasing in ability, then allowing vote buying is beneficial/harmful.

There are also some papers related to our methodological contribution. Bagnoli and Lipman (1988, henceforth BL) analyse a model in which a raider makes a takeover bid (that is not met by an incumbent's response). They develop a model with a finite number of shareholders and study its limit as the number grows. They contrast this with GH80 who analyse the same situation using a model with atomless and non-pivotal shareholders. BL do not define the asymmetric equilibria of the limit continuum game, and hence, they neither characterize nor study it directly as we do. Substantively, BL follow GH80 in inquiring how the free-rider problem might impede takeover attempts. Our substantive focus is instead on the effect of allowing trading of votes separately from shares in a contest. Hirshleifer and Titman (1990) develop a variant of GH80, based more on Shleifer and Vishny (1986), wherein the raider has private information and a block of shares (and the incumbent cannot respond to the raider's offer). Hirshleifer and Titman use asymmetric equilibria in a manner similar to what we do here to fully solve that model.

3. THE MODEL AND ANALYSIS

3.1. *The model*

This is a model of a contest for control of a firm. Initially, the firm is controlled by the incumbent management team, I, and the shares of the firm are spread uniformly across a continuum of identical shareholders denoted by the interval $[0,1]$. Each share is bundled with a vote. A rival management team, R, is trying to gain control of the firm by acquiring from the shareholders the majority of the votes. We will refer to R and I as the contenders.

Under R's control, the firm has value $\omega_R > 0$, which is the total value of the income accruing to the shareholders, and R has private benefit $b_R > 0$.[7] Similarly, ω_I and b_I represent the firm value and private control benefit under I's control. Thus, if in the end—after all transactions were performed and all contingencies realized—contender k owns a fraction α of the shares after having paid to shareholders the total sum of t, then *contender k's pay-off* is $\alpha\omega_k - t + b_k$ if it wins and it is $\alpha\omega_j - t$ if $j \neq k$ wins. When k wins, the *pay-off to a shareholder* who was paid z is $z + \omega_k$ if this shareholder still owns the share and just z if not.

To economize a bit on the taxonomy, we assume that $\omega_I + nb_I \neq \omega_R + n'b_R$, for any $n, n' \in \{0,1,2\}$. This implies in particular that in all scenarios the total value is always maximized under the control of a unique contender.[8]

We consider two basic situations with respect to the allowable trades: one where shareholders can tender only shares (bundled with the votes) and one where shareholders may also sell the

7. The assumption that the parties in control may be able to extract private benefits is standard in the related literature. Some theoretical justification is provided by Berle and Means (1932) and Jensen and Meckling (1976); some empirical justification can be found in Dodd and Warner (1983) and Johnson *et al.* (2000).

8. This assumption also guarantees that, when each contender makes the maximal offer it can make without incurring a loss, there will be no tie.

votes separately (while keeping the shares and hence the income accruing to them).[9] In the former, each contender $k \in \{I, R\}$ quotes a price p_k^{s} per share; in the latter, each quotes a pair of prices $(p_k^{\mathrm{s}}, p_k^{\mathrm{v}})$ for full shares (including votes) and for just votes (with no claim to income), respectively. In each of these situations, we consider three scenarios that differ in terms of the additional conditions that the contenders may attach to the price offers. In the basic scenario, the contenders are allowed to make only unrestricted price offers: all the shares tendered to them must be purchased at the quoted prices. In other scenarios, the contenders are allowed to qualify their price offers with quantity restrictions and conditions. We will present the details of those scenarios later on when we turn to analyse them. Since the basic model is common to all scenarios, we continue to outline the model using the general term "offer" to represent the combination of prices and whatever additional conditions that may accompany them in the different scenarios. Let F_k denote the set of feasible offers and $f_k \in F_k$ denote an individual offer, by contender $k \in \{I, R\}$.

The contenders move in sequence. First, R makes an offer $f_R \in F_R$ to all shareholders. Then I responds with an offer $f_I \in F_I$ to all shareholders. After observing both offers, shareholders make their tendering decisions simultaneously. Finally, R gains control if following the tendering stage R has successfully purchased 50% of the votes (either with or without shares). Otherwise I remains in control. In other words, the *status quo* is for I to remain in control unless R obtains more votes than I.[10]

Strategies are defined in the usual way. A strategy σ_R for R is a feasible offer, $\sigma_R \in F_R$; a strategy σ_I for I prescribes a feasible offer as a function of R's offer, $\sigma_I : F_R \to F_I$; a strategy for a shareholder specifies a tendering decision (whether and which of the offered tendering options to accept) as a function of the offers (f_R, f_I) made by R and I.

A *tendering outcome* is a four-tuple $m = (m_R^{\mathrm{s}}, m_R^{\mathrm{v}}, m_I^{\mathrm{s}}, m_I^{\mathrm{v}})$, where m_k^h is the fraction of all shares ($h = \mathrm{s}$) or votes ($h = \mathrm{v}$) that is being tendered to contender $k = R$ or I. (When only shares can be traded, $m_k^{\mathrm{v}} \equiv 0$ and we can write $(m_R^{\mathrm{s}}, m_I^{\mathrm{s}})$ instead.) The tendering outcome fully determines the fraction of votes that each of the contenders ends up controlling (*e.g.* in a scenario in which a contender must purchase all shares and votes tendered to it, R ends up controlling $m_R^{\mathrm{s}} + m_R^{\mathrm{v}}$ of the votes).

We denote by π *the probability that* R *wins*. The set of πs that are compatible with m is denoted by $\Pi(m)$. That is, if $m_R^{\mathrm{s}} + m_R^{\mathrm{v}} > 1/2$, then $\Pi(m) = \{1\}$; if $m_R^{\mathrm{s}} + m_R^{\mathrm{v}} < 1/2$, then $\Pi(m) = \{0\}$; and if $m_R^{\mathrm{s}} + m_R^{\mathrm{v}} = 1/2$, then $\Pi(m) = [0, 1]$.[11]

An *outcome* of the tendering subgame following offers f_R and f_I is a pair $(m, \pi)_{f_R, f_I}$ with $\pi \in \Pi(m)$.

3.2. *The solution concept*

3.2.1. Subgame perfect equilibrium. An *equilibrium in the tendering subgame* is an outcome $(m, \pi)_{f_R, f_I}$ satisfying the following: (i) If $m_k^h > 0$, for $h = \mathrm{s}$ or v and $k = I$ or R, then shareholders' expected pay-off from tendering instrument h to contender k is at least as

9. There is no need to consider the option of selling just the share without the vote since in the presence of risk neutrality and complete information assumed in this model, the value of a voteless share is the same for all actors and there is no reason to trade it.

10. The alternative where at the end of all trades there is a proxy vote is commented on later.

11. Letting any π be feasible when $m_R^{\mathrm{s}} + m_R^{\mathrm{v}} = 1/2$ will be necessary for the existence of an equilibrium in the tendering subgame.

Note that $m_R^{\mathrm{s}} + m_R^{\mathrm{v}} = 1/2$ and any π can arise as the limit behaviour as $N \to \infty$ over a sequence of models with N shareholders who tender to R with an appropriately chosen probability that tends to 1/2 while the winning probability it implies tends to π.

high as with any other available option. (ii) If some agent does not tender shares nor votes, *i.e.* $\sum_{k,h} m_k^h < 1$, then shareholders' expected pay-off from not tendering is at least as high as with any other available option.

For example, when only shares are traded, part (i) implies that

$$m_R^{\mathrm{s}} > 0 \Rightarrow p_R^{\mathrm{s}} \geq \max\{p_I^{\mathrm{s}}, \pi\,\omega_R + (1-\pi)\omega_I\},$$

while part (ii) means that

$$m_R^{\mathrm{s}} + m_I^{\mathrm{s}} < 1 \Rightarrow \pi\,\omega_R + (1-\pi)\omega_I \geq \max\{p_I^{\mathrm{s}}, p_R^{\mathrm{s}}\}.$$

We emphasize that π is determined in equilibrium: π enters the optimality conditions for shareholders, and π must also be consistent with shareholder behaviour ($\pi \in \Pi(m)$).

A subgame perfect equilibrium *(SPE) in the entire game*, given sets F_R, F_I of feasible offers, consists of strategies σ_k, $k = R, I$, and for each pair of offers f_R, f_I a selection of an equilibrium outcome in the tendering subgame $(m,\pi)_{f_R, f_I}$ such that neither R nor I can increase the pay-off it gets in the resulting outcome $(m,\pi)_{\sigma_R,\sigma_I(\sigma_R)}$ by deviating from its σ_k.

3.2.2. Our solution concept—a refinement of SPE.

Our solution concepts refines SPE by imposing two additional requirements. One rules out knife-edge equilibria that rely on shareholder indifference and would not survive perturbations of the game. The other essentially rules out equilibria in the subgame that are Pareto dominated for the shareholders. The formalization of these requirements is as follows.

Definition 1. The offers f_R, f_I are said to be *tie-free* if $p_k^h \neq p_j^h$ and $p_k^{\mathrm{s}} \neq p_j^{\mathrm{v}} + \omega_j$ for $h \in \{\mathrm{s},\mathrm{v}\}$ and $j \neq k \in \{R, I\}$.

Definition 2. An SPE $(f_R^*, \sigma_I^*, \{(m^*,\pi^*)_{f_R,f_I} : (f_R, f_I) \in F_R \times F_I\})$ is *robust* if for any f_R, f_I, and $\varepsilon > 0$, there are tie-free offers $(f_R^\varepsilon, f_I^\varepsilon)$ in an ε-neighbourhood of f_R, f_I and an equilibrium in the tendering subgame following $(f_R^\varepsilon, f_I^\varepsilon)$, denoted $(m,\pi)_{f_R^\varepsilon, f_I^\varepsilon}$, such that

1. $\left|(m^*,\pi^*)_{f_R,f_I} - (m,\pi)_{f_R^\varepsilon, f_I^\varepsilon}\right| < \varepsilon$ and
2. $(m,\pi)_{f_R^\varepsilon, f_I^\varepsilon}$ is not Pareto dominated for the shareholders by any strict equilibrium in the tendering subgame following $f_R^\varepsilon, f_I^\varepsilon$.

In other words, consider the outcome $(m^*,\pi^*)_{f_R,f_I}$ prescribed by the equilibrium for the tendering subgame following the offers (f_R, f_I). If these offers involve no ties and there is no other strict equilibrium outcome that is preferred by all shareholders, then the robustness condition is satisfied. If (f_R, f_I) involve ties, then the robustness condition requires that there must be nearby offers, $(f_R^\varepsilon, f_I^\varepsilon)$, that involve no ties and such that there is some equilibrium outcome $(m,\pi)_{f_R^\varepsilon, f_I^\varepsilon}$ of the ensuing subgame that is (1) close to the original equilibrium $(m^*,\pi^*)_{f_R,f_I}$ and (2) not Pareto dominated by any strict equilibrium of that subgame.

Part (1) of the robustness refinement pins down how ties are broken.[12] In its absence, tie breaking will not be pinned down uniquely by the equilibrium. For example, consider the scenario in which the contenders may only buy shares at unrestricted prices. Consider a subgame after R offers a price $p_R^{\mathrm{s}} \in (\omega_I + b_I, \omega_I + 2b_I)$. If I were to offer $p_I^{\mathrm{s}} = p_R^{\mathrm{s}}$, then shareholders would be indifferent between tendering to I and to R. Then I would profit from this if a bit

12. The definition of tie-free offers is stated here only in terms of uncontingent prices p_k^{s} and p_j^{v} since we have not yet introduced the notation for contingent offers. But it will apply to them in the same way as we will note again after introducing the required notation in Section 6.

more than 50% of the shareholders would tender to it, but I would suffer losses if all shareholders would tender to it. Thus, in this subgame, there are multiple equilibria that differ in how shareholders break ties when they are indifferent. This observation distinguishes this model from some other Bertrand-style models in which tie breaking is uniquely determined in equilibrium. The robustness requirement rules out equilibria of the form just mentioned that are clearly knife-edge. It implies, *e.g.* that in the equilibrium of the subgame following the offers $p_I^s = p_R^s$, the shareholders will not tender both to R and to I. This follows from (1) because for any nearby tie-free offers $p_k^\varepsilon \approx p_k$, we have $p_I^\varepsilon \neq p_R^\varepsilon$ and then the unique equilibrium in the tendering subgame following $(p_I^\varepsilon, p_R^\varepsilon)$ has shareholders tendering to the contender offering the higher price and not to both.

To understand our motivation for (2), note that, as is common in voting games, inefficiencies in our model can arise due to coordination failures. Since our purpose is to focus on the inefficiencies due to the trading rules—in particular whether votes can be sold separately—we adopt a refinement that rules out inefficiencies that arise due to coordination failures.

Henceforth, when we refer to an *equilibrium* of the game we mean a *robust SPE* (except of course when we explicitly refer to SPE or to (Nash) equilibria of the tendering subgame).

3.3. *Overview of the analysis*

The analysis focuses on the contrast between the case where votes can be traded separately and the case where they cannot. As mentioned above, this comparison is conducted in three different scenarios with respect to the nature of the offers that the contenders may make. The structure of all the cases, however, is similar and goes as follows.

Section 8 establishes that in all scenarios there exists an equilibrium. In every case, we show that there cannot be an equilibrium in which $\pi \in (0, 1)$. The conclusion from these two observations is that, in equilibrium, one of the contenders wins with certainty ($\pi = 1$ or $\pi = 0$). It is then relatively straightforward to rule out one of these possibilities, thereby identifying the equilibrium winner for each configuration of the parameters.

This allows us to draw conclusions regarding the overall efficiency of the equilibrium. By our definition, the outcome is *efficient* if the contender that generates the maximal total value, $w_k + b_k$, wins. We then also use these observations, combined with some properties of the contenders' best replies, to comment on the pay-offs that shareholders receive in equilibrium.

Throughout the analysis we stick to the basic scenario outlined above where R must gain control over at least 50% of the votes in order to win. In the Appendix, we also present results for an alternative scenario in which the contest ends with a vote. Allowing for voting at the end changes the game because then R does not need to purchase a majority of the votes to obtain control, it is enough that R obtains a majority in the vote at the end. However, the main results are unchanged.

Despite the similarity in the general structures of the proofs, every scenario requires some specialized work, so it is not possible to provide a unified proof. Still to help the reading, we present in the body of the paper only the proofs of the first (and simplest) scenario. The proofs for the remaining cases are relegated to the Appendix.

4. UNRESTRICTED AND UNCONDITIONAL OFFERS

In this section, we consider the simplest trading rule. The contenders' price offers cannot be quantity constrained—they must purchase the entire quantities tendered to them at the prices they quote. The main results of this section are that, when votes cannot be traded separately, the equilibrium outcome is efficient (maximizes $w_k + b_k$), and with vote trading, it need not be efficient. We characterize precisely when inefficiency arises if vote trading is allowed. Roughly speaking, the "wrong" contender can win when its private benefits are sufficiently larger than those of the other contender; vote trading enables it to win even when it is not efficient.

4.1. *Only shares*

In this subsection, votes are inseparable from shares. So, a feasible offer by contender $k = R, I$ is a price p_k^s at which it must purchase all shares tendered to it. To gain control, R must purchase at least 50% of the shares.

Theorem 1. *The contender with the higher total value, $w_j + b_j$, wins in all equilibria.*

Proof. Follows from the following two lemmas. ||

Lemma 1. *There is no equilibrium in which both contenders win with strictly positive probability, i.e. there is no equilibrium with $\pi \in (0,1)$.*

Proof. Robustness implies that, in any equilibrium, it cannot be that some shareholders sell some shares to I and some to R because any tie-free offers near (p_R^s, p_I^s) will break the indifference and change the outcome discontinuously. So, if $\pi \in (0,1)$ arises at equilibrium, it must be that half the shareholders tender to R and half do not tender at all. Hence, $p_I^s \leq p_R^s$, and those who do not tender to R hold out to get the expected value $\pi w_R + (1-\pi)w_I$. In such a case

$$p_R^s = \pi w_R + (1-\pi)w_I, \tag{1}$$

for otherwise all shareholders would tender either to R or not at all. Finally, it must also be that $w_I \leq p_R^s$ since if $w_I > p_R^s$ this equilibrium would fail the Pareto part of robustness since its outcome (and any sufficiently close outcome) would be dominated by a strict equilibrium in the tendering subgame in which shareholders do not tender at all.

Let u_j denote the profit of $j = I, R$ in the equilibrium with $\pi \in (0,1)$.

$$u_I = (1-\pi)b_I, \tag{2}$$

$$u_R = \frac{1}{2}[-p_R^s + \pi w_R + (1-\pi)w_I] + \pi b_R \tag{3}$$

$$= \pi b_R \text{ (by equation (1))}.$$

1. Suppose $w_I + b_I > w_R + b_R$.

   Consider an equilibrium in which $\pi \in (0,1)$. Let $\hat{u}_I$ denote I's profit after offering p_I^s just above p_R^s. Since $p_I^s > p_R^s \geq w_I$, all shareholders will tender to I. Choosing p_I^s in the interval $(p_R^s, p_R^s + \pi[w_I + b_I - w_R - b_R])$, we get

$$\hat{u}_I = -p_I^s + w_I + b_I$$
$$> -p_R^s + \pi(w_R + b_R) + (1-\pi)(w_I + b_I)$$
$$= \pi b_R + (1-\pi)b_I \geq (1-\pi)b_I = u_I, \tag{4}$$

   where the second equality holds by the equilibrium condition (1). Thus, I can deviate profitably from the putative equilibrium with $\pi \in (0,1)$.

2. Suppose $w_I + b_I < w_R + b_R$.

   Since this is an equilibrium, I cannot profitably outbid R with p_I^{s} just above p_R^{s}. That is,

$$u_I \geq b_I + w_I - p_R^{\text{s}} \tag{5}$$

$$= b_I + w_I - [\pi w_R + (1-\pi)w_I]$$

$$= (1-\pi)b_I + \pi(w_I + b_I - w_R), \tag{6}$$

   where the first equality follows from equation (1). If $w_I + b_I > w_R$, then $u_I > (1-\pi)b_I$ in contradiction to equation (2). If $w_I + b_I < w_R$, then $\pi \in (0, 1)$ may not arise in equilibrium since $p_R^{\text{s}\prime} = w_R$ would guarantee R a win with profit $b_R > \pi b_R = u_R$ in contradiction to the equilibrium hypothesis.[13] ‖

Lemma 2. *If $b_I + w_I < b_R + w_R$, then $\pi = 0$ cannot occur in equilibrium; if $b_I + w_I > b_R + w_R$, then $\pi = 1$ cannot occur in equilibrium.*

Proof. Suppose first that $w_I + b_I > w_R + b_R$. It cannot be that $\pi = 1$. If $p_R^{\text{s}} > w_R + b_R$ and $\pi = 1$, then all shareholders tender to R and R has a loss. So, since R's profitability implies $p_R^{\text{s}} \leq w_R + b_R$, I can win profitably with p_I^{s} just above $w_R + b_R$. Suppose next that $w_I + b_I < w_R + b_R$. If $b_R > 0$, then $p_R^{\text{s}} > \max\{w_I + b_I, w_R\}$ would guarantee profitable win for R, which I can defeat only at a loss, while if $b_R = 0$, then $p_R^{\text{s}} \in (w_I + b_I, w_R)$ (which is a non-empty interval) guarantees a profitable win for R which I can defeat only at a loss. ‖

In terms of shareholder payments, the equilibrium outcome is not necessarily unique. If $w_I + b_I > w_R + b_R$, then I always wins but there are multiple equilibria as R's behaviour can impact pay-offs to I and to shareholders. Specifically, depending on R's initial move, shareholder pay-offs could range anywhere in $[w_I, w_I + b_I]$. (However, equilibria with pay-offs above $w_R + b_R$ involve weakly dominated offers by R.) If $w_I + b_I < w_R + b_R$, then shareholders pay-offs are $\max\{w_I + b_I, w_R\}$.

4.2. *Both votes and shares*

In this scenario, votes can be traded separately from shares. The contenders' offers take the form $(p_j^{\text{s}}, p_j^{\text{v}})$, where p_j^{s} is the price for the full share (including its vote) and p_j^{v} is the price per vote offered by $j = R, I$. As above, contenders are committed to purchase any quantities tendered to them.

In this case, vote trading interferes with efficiency: the winner is not always the efficient contender (the maximizer of $w_j + b_j$). To gain some intuition, recall that when votes cannot be traded and $w_R + b_R > w_I + b_I$, R wins with $p_R^{\text{s}} = w_I + b_I$ even if $b_I > b_R$.

We now argue that, when votes can be traded and b_I is large enough, then R cannot win profitably with any bid for shares $p_R^{\text{s}} \leq w_R + b_R$. For simplicity of this intuitive description, we assume that the incumbent I provides only private benefits, $b_I > 0$, $w_I = 0$, and the rival R only shareholder benefits, $b_R = 0$, $w_R > 0$, and suppose that $w_R > b_I$. Clearly, R cannot win with a bid for shares that is less than b_I as then I can simply overbid and profitably win. So consider an

13. This argument would fail if $b_R = 0$. In that case, there are multiple equilibria, where R can offer any price $p_R^{\text{s}} \in [w_I + b_I, w_R]$ and win with probability $\pi \in [b_I/(w_R - w_I), 1]$, and in all these equilibria, R obtains 0 profits.

offer for shares by R that lies between b_I and w_R. The important point is that, although R's offer is above b_I, it may still be possible for I to profitably win. The key to this observation is that, after I responds with an offer for votes just below R's bid for shares, there is only an asymmetric equilibrium in the tendering subgame, so that I will buy only half the votes and hence can afford to offer more than b_I per vote.[14]

To see this, note that if the majority of shareholders tender their shares to R, then any shareholder is better off tendering his vote to I as this will give him the vote's price plus w_R (that is obtained when R is in control). If instead the majority tender their votes to I, then an individual shareholder knows that the share value will be zero and hence he is better off tendering to R for the offered share price that is higher than the vote price offered by I. Thus, in the only equilibrium in such subgame, shareholders randomize equally between the two (since there is a large number of shareholders, they must randomize equally for the outcome to be stochastic), and hence, I buys only half the votes.

The next important feature of this offer is that I can win with probability close to 1 in the asymmetric equilibrium of the tendering subgame. To see this, recall first that for shareholders to behave asymmetrically, they must be indifferent. If they sell to R, they get R's bid for shares, p_R^s, while if they sell to I, they get I's bid for votes, p_I^v, plus the share value of w_R if and only if R wins (since the share value under I is zero). If p_I^v is just below p_R^s, then for these to be equal the probability of R winning must be close to zero.

Hence, with this unrestricted offer for votes, I is able to achieve the equivalent of a restricted offer. This enables I to profitably bid for votes so long as buying half the votes at (just below) R's total value of w_R is worthwhile, *i.e.* so long as $2b_I > w_R$. We have thus seen that—*because of the asymmetric equilibria that arise in tendering subgames*—R cannot win even when the total value that R provides, w_R, is greater than I's total value of b_I.

Now note that in the equilibrium of the subgame just described, R is purchasing half the shares at a positive price and I is obtaining control with probability 1, so R's purchase is not profitable, and hence, R would not initially make such an offer, leaving I in control.

Theorem 2. *The efficient contender wins in equilibrium except in the following regions of the parameter space.*

1. *If $w_I + b_I > w_R + b_R$ and $b_R > 2b_I$, then R wins though I is the efficient contender.*
2. *If $w_I + b_I < w_R + b_R < w_I + 2b_I$ and $b_I > b_R$, then I wins though R is the efficient contender.*

The proof is in the Appendix. The method is as before. It is first shown that there are no asymmetric equilibria in which both contenders win with positive probability. Then for each region of the parameter space, one of the contenders is eliminated as a possible winner, which leaves the other as the sole candidate for winning. Since existence is assured, this characterization implies the result.

4.2.1. First- and second-mover advantages. The characterization in Theorem 2 reflects both a first-mover and a second-mover advantage.

- *Second-mover advantage:* When $w_R + b_R$ is not too much larger than $w_I + b_I$, then I can win with even a small advantage in private benefits, $b_I > b_R$. By contrast, if $w_I + b_I >$

14. The term asymmetric strategies (or equilibrium) means throughout that different shareholders make different tendering decisions. Since shareholders are identical, this is a purification of a symmetric mixed strategy. But since there is a continuum of shareholders, it is more straightforward to talk about asymmetric than about mixed.

$w_R + b_R$ and R's advantage in private benefits is not too large, $b_I < b_R < 2b_I$, then I wins. So, those situations exhibit a second-mover advantage.

The source of the second-mover advantage is in the ability to make an offer that induces an asymmetric equilibrium in the tendering subgame in which the second mover acquires just half the shares or votes. This enables the second mover to offer a premium above the true value. The first mover cannot do so for fear of having to pay the premium to all shareholders. So, the second mover can effectively mimic the effect of a quantity restriction even when it cannot be explicitly imposed.

- *First-mover advantage:* When $b_R > 2b_I$, R wins regardless of how much greater is $w_I + b_I$ relative to $w_R + b_R$. In contrast, when $b_I > 2b_R$, then I would still lose if $w_R + b_R > w_I + 2b_I$. So, in those situations there is a first-mover advantage.

  The source of the advantage is R's ability to make a pre-emptive offer to buy votes. Even when w_I is far greater than w_R, beating such a pre-emptive offer would result in a loss for I. The fact that such a response would result in a loss for R as well does not help I since R's offer is already in place. For R's pre-emptive offer to be successful, b_R must be more than twice b_I. This is because I can again use its second-mover ability to induce an asymmetric equilibrium in which it buys only half the shares and hence can offer premium of up to $2b_I$ over their public value.

  More specifically, if $b_R > 2b_I$ and $w_I + b_I > w_R + b_R$, then I cannot win profitably following an initial offer by R of $p_R^{\text{v}} = 2b_I + \varepsilon$. Obviously, I cannot win profitably with $p_I^{\text{v}} \geq p_R^{\text{v}}$. Consider then I's possible responses with p_I^{s}. If $p_I^{\text{s}} < w_R + 2b_I + \varepsilon$, then all shareholders sell to R so I will lose. If $p_I^{\text{s}} \in [w_R + 2b_I + \varepsilon, w_I + b_I)$, then since $w_I > w_R$ (which follows from $b_R > 2b_I$ and $w_I + b_I > w_R + b_R$), in the equilibrium of the ensuing subgame I cannot win with probability 1. (This is because, if I wins at $p_I^{\text{s}} < w_I + b_I$, then an individual shareholder does better selling to R and earning $w_I + 2b_I + \varepsilon$.) Thus, either R wins or it is an asymmetric equilibrium in which half sell to R and half to I. The latter requires indifference, $p_I^{\text{s}} = \pi w_R + (1-\pi)w_I + p_R^{\text{v}}$, and then I's profits are $(1-\pi)b_I + ((\pi w_R + (1-\pi)w_I) - p_I^{\text{s}})/2 = (1-\pi)b_I - b_I < 0$ (the expected benefit of control plus the loss on the shares acquired by I which are half of the total).

  In contrast, when $b_I > 2b_R$ and $w_R + b_R > w_I + 2b_I$, I cannot win profitably. In this case, R can offer to buy shares at $p_R^{\text{s}} = w_I + 2b_I + \varepsilon$ against which I has no profitable response. Again it is obvious that no offer p_I^{s} for shares can be beneficial to I. An offer with $p_I^{\text{v}} < 2b_I$ attracts no shareholders, while an offer of $p_I^{\text{v}} > 2b_I$ induces an equilibrium in the subgame with shareholders tendering to both in which I's profit is negative: $(1-\pi)b_I - p_I^{\text{v}}/2 < 0$.

The reader might be concerned that the inefficiency here owes to the specific extensive form assumed in the model. First, the above discussion clarifies that the inefficiency may arise with any order of moves. The specific order might affect the region of the parameter space at which the inefficiency will arise, but the qualitative observation that the separation of votes from shares may undermine the efficiency remains valid in all cases and the fundamental explanations are of the same nature. Second, it is also easy to see that the inefficiency is not an artifact of the finite horizon. At least some of the inefficient equilibria are also subgame perfect equilibria of the infinite-horizon game.[15] For example, consider an equilibrium of Type 1 where R is the inefficient contender that wins with an initial offer to which I has no profitable response. Obviously, this is also an SPE outcome in the infinite-horizon game. (If I had a profitable response to which R could not profitably respond when R can make a further counter-offer, then I could certainly

15. We do not comment on robust equilibria on which we focus elsewhere in this paper, as the definition would have to be suitably modified and existence re-established, and that goes beyond the scope of this paper.

make this response in the current game.) Finally, it is important to remember that the order and the sequential nature of the bidding emerge naturally from the scenarios that are being modelled here. It is therefore not surprising that much of the related literature has adopted this extensive form and even just for the sake of comparison with the existing literature it makes sense to retain it.

4.2.2. Shareholder profits. We also examine the effect of vote trading on shareholders' pay-offs. The comparison of pay-offs across the different regimes is sometimes ambiguous due to the presence of multiple equilibria: when I wins in equilibrium, the pay-offs to I and to the shareholders depend on R's initial actions and R is indifferent among a wide range of actions. However, just like the conclusions of GH88 for dual-class shares, even when the comparison is unambiguous it can go either way: the introduction of separate vote trading sometimes enhances and sometimes harms shareholders pay-offs.

For example, when $\omega_I + b_I > \omega_R > \omega_I$ and $b_R > \min\{\omega_I - \omega_R + 2b_I, b_I\}$, contender R wins whether or not votes can be traded separately, but shareholders pay-offs with vote trading ($\min\{\omega_I + 2b_I, \omega_R + b_I\}$) are larger than without it ($\omega_I + b_I$). The intuition behind this observation is that vote trading benefits the shareholders because it forces R to make a more aggressive offer. When votes cannot be traded, for R to win it must offer $p_R^{\mathrm{s}} = \omega_I + b_I$. When votes can be traded, if R simply offers $p_R^{\mathrm{s}} = \omega_I + b_I$, then I can respond with $p_I^{\mathrm{v}} = b_I - \varepsilon$ and, for sufficiently small ε, will win profitably with probability close to 1 (the equilibrium in the tendering subgame following these offers is asymmetric). Therefore, R must either offer $p_R^{\mathrm{s}} = \omega_I + 2b_I$ or $p_R^{\mathrm{v}} = b_I$ to deter I, both of which lead to higher pay-offs to shareholders.

By contrast, when $\omega_R < \omega_I$ and $b_R > \omega_I + b_I - \omega_R > 2b_I$, contender R wins whether or not votes can be traded separately, but shareholders pay-offs with vote trading ($\omega_R + 2b_I$) are smaller than without it ($\omega_I + b_I$). This is because in the absence of vote trading, R has to offer $p_R^{\mathrm{s}} = \omega_I + b_I$, while with vote trading it can win with buying just votes at $p_I^{\mathrm{v}} = 2b_I$.

Thus, vote trading can benefit shareholders because it may force R to make a more aggressive initial offer when faced with the possibility of subsequent offers for votes. It can be harmful under other parameters because R may win control by buying only votes at a lower price than if R had to buy shares.

5. RESTRICTED OFFERS

The change from the previous analysis is that the contenders are allowed to make restricted offers that cap the quantities of shares and/or votes that they will buy at the prices they announce. That is, a contender is committed to buy at the price it announced any quantity tendered to it up to the pre-announced quota. Intuitively, it seems that such a cap should enable contenders to offer higher premiums over the public value of the shares since by capping the quantity they would not have to pay this premium to all shareholders. It therefore should bias the outcome in favour of contenders with higher private benefits. This type of result appears in GH88 and subsequent literature and is also confirmed by the following analysis. Note though that while the direction of the bias is the same as in the case of allowing vote buying, the cases in which inefficiency occurs differ.

5.1. *Only shares*

First consider the case in which votes can be transferred only by trading shares. As before, the rival has to acquire a majority of the shares to take control. An offer f_j by contender $j = R, I$ is a pair $f_j = (p_j^{\mathrm{s}}, \overline{m}_j^{\mathrm{s}})$. This is a commitment to buy at the price p_j^{s} any quantity tendered to it up to $\overline{m}_j^{\mathrm{s}}$. Recall that the outcome of the ensuing tendering subgame is $(m_R^{\mathrm{s}}, m_I^{\mathrm{s}}; \pi)$, where m_j^{s} is the mass of shareholders who decide to tender to $j = R, I$ and π is the probability that R wins. If $\overline{m}_j^{\mathrm{s}} < m_j^{\mathrm{s}}$, then the m_j^{s} shareholders who tendered to j are rationed with equal probability and only a fraction $\overline{m}_j^{\mathrm{s}}/m_j^{\mathrm{s}}$ end up tendering.

Thus, if $(m_R^{\mathrm{s}}, m_I^{\mathrm{s}}; \pi)$ is an equilibrium outcome of the tendering subgame, it must satisfy the following conditions:

- If $m_j^{\mathrm{s}} > 0$, then tendering to j should be at least as beneficial as the alternative options of tendering to the other bidder or keeping the share. That is,

$$\min\left\{\frac{\overline{m}_j^{\mathrm{s}}}{m_j^{\mathrm{s}}}, 1\right\} p_j^{\mathrm{s}} + \left[1 - \min\left\{\frac{\overline{m}_j^{\mathrm{s}}}{m_j^{\mathrm{s}}}, 1\right\}\right] \times [\pi\omega_R + (1-\pi)\omega_I]$$

$$\geq \max\left\{\min\left\{\frac{\overline{m}_j^{\mathrm{s}}}{m_j^{\mathrm{s}}}, 1\right\} p_{-j}^{\mathrm{s}} + \left[1 - \min\left\{\frac{\overline{m}_j^{\mathrm{s}}}{m_j^{\mathrm{s}}}, 1\right\}\right] \times [\pi\omega_R + (1-\pi)\omega_I],\right.$$

$$\left.\pi\omega_R + (1-\pi)\omega_I\right\}.$$

  Here $\min\{(\overline{m}_j^{\mathrm{s}}/m_j^{\mathrm{s}}), 1\}$ is the proportion of shareholders who offer their shares to j and succeed in selling them. These shareholders obtain p_j^{s}, while the others receive $\pi\omega_R + (1-\pi)\omega_I$. The max is over the option of offering one's share to $-j$ and not tendering at all.
- If $m_R^{\mathrm{s}} + m_I^{\mathrm{s}} < 1$, then the option of not tendering is at least as beneficial as tendering. That is, for each $j = R, I$,

$$\pi\omega_R + (1-\pi)\omega_I \geq \min\{(\overline{m}_j^{\mathrm{s}}/m_j^{\mathrm{s}}), 1\} p_j^{\mathrm{s}} + [1 - \min\{(\overline{m}_j^{\mathrm{s}}/m_j^{\mathrm{s}}), 1\}][\pi\omega_R + (1-\pi)\omega_I].$$

Remark 1. We specify that if $\bar{m}_R^{\mathrm{s}} = 1/2$ and $m_R^{\mathrm{s}} > 1/2$, then R wins.

The main intuition of the following analysis is that, since the winning contender can cap its offer at half the shares, it can bid up to $\omega_j + 2b_j$ and still break even. Therefore, we expect that I wins if $\omega_I + 2b_I > \omega_R + 2b_R$ and R wins if the reverse inequality holds strictly.

Theorem 3. *In all equilibria, the contender with the higher value of $\omega_j + 2b_j$ wins.*

The proof is in the Appendix and again follows the logic of first ruling out equilibria with $\pi \in (0, 1)$.

5.2. *Both votes and shares*

An offer f_j by $j = R, I$ is a four-tuple $f_j = (p_j^{\mathrm{s}}, \overline{m}_j^{\mathrm{s}}; p_j^{\mathrm{v}}, \overline{m}_j^{\mathrm{v}})$, where p_j^{s} and p_j^{v} are the prices offered by j for shares and votes, respectively, while $\overline{m}_j^{\mathrm{s}}$ and $\overline{m}_j^{\mathrm{v}}$ are the respective quantity restrictions. The main result here is that vote buying harms efficiency in the sense that the region of the parameter space over which the efficient contender wins shrinks in comparison to the case in which votes cannot be traded separately.

An outcome of the tendering subgame following f_R and f_I is $(m, \pi)_{f_R, f_I} = (m_R^{\mathrm{s}}, m_R^{\mathrm{v}}, m_I^{\mathrm{s}}, m_I^{\mathrm{v}}; \pi)_{f_R, f_I}$, where m_j^{s} and m_j^{v} are the masses of shareholders who decide to tender shares and votes, respectively, to $j = R, I$ given offers (f_R, f_I) and as before π is the probability that R wins following these offers. The rationing rules are as before and are applied to each offer separately. If $\overline{m}_j^{\text{s}} < m_j^{\text{s}}$, only a fraction $\overline{m}_j^{\text{s}}/m_j^{\text{s}}$ end up tendering shares to j, and similarly if $\overline{m}_j^{\text{v}} < m_j^{\text{v}}$, only a fraction $\overline{m}_j^{\text{v}}/m_j^{\text{v}}$ end up tendering votes to j, independently of contender j's other offer. At such an outcome, the expected pay-off of tendering shares to j is $\min\{(\overline{m}_j^{\text{s}}/m_j^{\text{s}}), 1\}p_j^{\text{s}} + [1 - \min\{(\overline{m}_j^{\text{s}}/m_j^{\text{s}}), 1\}][\pi\omega_R + (1-\pi)\omega_I]$; the expected pay-off of tendering votes to j is $\min\{(\overline{m}_j^{\text{v}}/m_j^{\text{v}}), 1\}p_j^{\text{v}} + [\pi\omega_R + (1-\pi)\omega_I]$. In an equilibrium of the tendering subgame, any action taken by a positive mass of shareholders (tendering shares and/or votes or not tendering at all) must yield to shareholders expected pay-off at least as high as the expected pay-off of any of the available options of tendering or not.

Remark 2. As in Remark 1, if R is oversubscribed when it restricts its purchases to half the shares and votes, then it wins. That is, if $\min\{\bar{m}_R^{\text{v}}, m_R^{\text{v}}\} + \min\{\bar{m}_R^{\text{s}}, m_R^{\text{s}}\} = 1/2$ and $m_R^{\text{s}} > \bar{m}_R^{\text{s}}$ or $m_R^{\text{v}} > \bar{m}_R^{\text{v}}$, then R wins.

Theorem 4. *The identity of the winner is the same as in Theorem 3 except for parameter configurations satisfying $\omega_I + 2b_I > \omega_R + 2b_R$ and $b_R > b_I$. For these configurations, I is the efficient contestant and would be the winner in the absence of vote trading, but R wins when vote trading is allowed.*

The proof is in the Appendix and its logic is again as in previous cases. It is argued first that in all equilibria $\pi \notin (0, 1)$. Then for each region of the parameter space, either $\pi = 0$ or $\pi = 1$ is ruled out which implies (via existence) that the remaining case prevails in equilibrium.

5.2.1. First- and second-mover advantages. The results above show that with restricted offers, there is only a first-mover advantage (and no second-mover advantage). This is consistent with the reason for the second-mover advantage when restricted offers are not possible. There we argued that the second-mover advantage results from the ability of the second mover to create an asymmetric equilibrium in the tendering subgame in which the second mover obtains half the votes but that the first mover cannot do so for fear of having to pay all the shareholders. With the ability of making restricted offers, this limitation on the first mover does not exist, and the first mover can do exactly what the second mover achieves. Indeed, the first mover, R, wins with restricted offers in strictly more cases than R does when R cannot make restricted offers.

6. CONTINGENT OFFERS

In this scenario, contenders are allowed to make contingent offers, an offer that takes effect if and only if the offering contender wins. An offer by contender $k = I, R$ for shares is a pair of prices: a contingent price p_k^{sc} at which contender k will buy all shares that were tendered to it in the event that it wins and a non-contingent price p_k^{s} at which it is committed to buy in any case. Similarly, an offer by contender $k = I, R$ for votes specifies a contingent price p_k^{vc} and a non-contingent price p_k^{v}. Each of these prices stands for a contender's commitment to purchase any quantity tendered subject to the contingency.

Now that we have the notation, we restate Definition 1 of tie-free offers to apply to contingent offers as well: The offers f_R, f_I are tie-free if $p_k^h \neq p_j^h$ and $p_k^{\text{s}} \neq p_j^{\text{v}} + \omega_h$ for $h \in \{\text{s}, \text{v}, \text{sc}, \text{vc}\}$ and $j \neq k \in \{R, I\}$.

6.1. *Only shares*

Again we first consider the case in which only shares can be traded. An outcome of the tendering subgame is an array of the form $(m_R^{s}, m_R^{sc}, m_I^{s}, m_I^{sc}, \pi)$. Thus, the offers are unrestricted offers but they can be conditioned on winning. The main result here is that outcome is efficient—the contender with the highest $w_k + b_k$ wins—as in the case of non-contingent and unrestricted offers for shares alone. Thus, unlike quantity restrictions this form of contingency does not interfere with efficiency.

Theorem 5. *If $w_k + b_k > w_j + b_j$, then in all equilibria k wins.*

The proof is in the Appendix and its method is again to rule out asymmetric equilibria in which both contenders win with positive probability. We know from the analysis in Section 4.1 that there is no such equilibrium when both contenders make non-contingent offers. This conclusion is extended here to the cases in which at least one contender makes a conditional offer.

6.2. *Both votes and shares*

Now allow for votes to be traded separately. Here, an outcome of the tendering subgame is an array of the form $(m_R^{s}, m_R^{sc}, m_R^{v}, m_R^{vc}, m_I^{s}, m_I^{sc}, m_I^{v}, m_I^{vc}, \pi)$. The analysis is similar to the case with non-contingent unrestricted offers. While more complicated as there are more cases to consider, surprisingly the outcome is unaffected by allowing for contingent offers.

Theorem 6. *The efficient contender wins in equilibrium except in the following regions of the parameter space.*

1. *If $w_I + b_I > w_R + b_R$ and $b_R > 2b_I$, then R wins.*
2. *If $w_I + b_I < w_R + b_R < w_I + 2b_I$ and $b_I > b_R$, then I wins.*

The proof is in the Appendix and the argument follows the same logic of ruling out asymmetric equilibria as in the previous proofs.

7. VARIATIONS ON THE BASIC MODEL: VOTING IN THE END

In the version of the model analysed so far, R gains control only if it acquires more than 50% of the votes. In an alternative description of the process, the bidding contest is followed by a vote that determines which contender will end up in control. In such a case, R might gain control even when it does not acquire the majority of the votes. It is not entirely clear which is the “right” model. Some related contributions in the finance literature employ the former model and some employ the latter. The rationale for using the model without the voting in the end is that to force a vote on control the raider might have to acquire a majority of the votes.

However, this question is not important for our conclusions regarding efficiency since the introduction of voting to the model would not change the results. To see this, consider a modified version of the model with voting in the end. That is, once the tendering stage is over, the two contenders with the blocks they have acquired and the remaining shareholders (who have sold neither their vote nor share) vote and the contender who wins this vote gains control. We will establish the claim by showing that any equilibrium outcome in the voting version has an equivalent outcome with the same winning probabilities in the game without voting.[16] We present the argument for the environments in which the contenders can make unrestricted offers for shares or for both shares and votes. It is clear that the argument can be extended to the case of restricted offers as well, but this will require some additional steps and we will forgo it here.

Observe first that, if $w_R < w_I$, those who do not tender to R end up voting for I, so in order to win R must still acquire over 50% of votes and nothing changes in the above analysis. Consider, therefore, the case of $w_R > w_I$ and a particular equilibrium in this case. Let π denote the probability with which R wins and θ_k denote the fraction of the total votes (with or without shares) that $k = R, I$ ends up purchasing in this equilibrium. Clearly, if $\theta_R > 1/2$, this equilibrium is automatically an equilibrium in the absence of voting as well. Similarly, if $\pi = 0$, this is also the case, since if R cannot deviate profitably when there is voting in the end, it cannot do so in the absence of voting. Finally, if $\pi > 0$ and $\theta_R \leq 1/2$, consider a configuration that differs from the equilibrium configuration only in that R offers an unrestricted price for shares $p^{\mathrm{s}}_R = \pi w_R + (1-\pi) w_I$ (*i.e.* the other parts of R's offer and those of I's offer are just as in the equilibrium); all the shareholders who tender shares to R or vote for R in the equilibrium sell shares to R at this p^{s}_R and all other shareholders behave as in the equilibrium. It can be verified that this configuration is an equilibrium outcome in the game without voting in the end. The shareholders who sell shares to R at p^{s}_R get the same pay-off as those voting for R in the equilibrium and so do the shareholders who sell to I or to another part of R's offer. Both R and I get the same pay-offs. Clearly, R does not have a profitable deviation since it would be available in the equilibrium with voting as well. Similarly, any profitable deviation by I would have the same effect in the equilibrium with voting. Thus, the constructed configuration is an equilibrium configuration in the game without voting.

8. EXISTENCE

In this section, we prove existence of an equilibrium. The method is to consider limits of equilibria of a sequence discretized games (where the actions spaces of I and R are finite, and there is a continuum of shareholders). The grids for the discretized games are selected so as to preclude ties (*i.e.* in our terminology, any pair of offers in a discretized game is "tie-free").

Recall the notation f_j, $j = I, R$, is an offer, F_j is the set of feasible offers for j, and an outcome in the tendering subgame following (f_R, f_I) is a tuple of the form $(m,\pi)_{f_R, f_I} = (m^{\mathrm{s}}_R, m^{\mathrm{v}}_R, m^{\mathrm{s}}_I, m^{\mathrm{v}}_I; \pi)_{f_R, f_I}$ consisting of the fractions of shareholders tendering shares and votes to each firm, and the probability π with which R wins. Let $C(f_R, f_I)$ denote the set of equilibrium outcomes in the tendering subgame which are not Pareto dominated by a strict equilibrium outcome in the tendering subgame. Let $u_j(f_R, f_I, (m,\pi)_{f_R, f_I})$ denote the pay-off to contender j given f_R, f_I and an outcome $(m,\pi)_{f_R, f_I}$ in the subgame following (f_R, f_I). Finally, let $U_j(f_R, f_I) = \{u_j(f_R, f_I, (m,\pi)) \colon (m,\pi) \in C(f_R, f_I)\}$.

F_j varies across the different scenarios as follows:

- In the unrestricted-shares case, $F_j = \mathbf{R}_+$ is a set of p^{s}-s (prices for shares).
- In the case of unrestricted shares and votes, $F_j = \mathbf{R}^2_+$ is a set of $(p^{\mathrm{s}}, p^{\mathrm{v}})$ pairs (prices for shares and for votes).
- In the quantity-restricted shares case, $F_j = \mathbf{R}_+ \times [0,1]$ is a set of $(p^{\mathrm{s}}, \overline{m}^{\mathrm{s}})$ pairs (share price and quantity restriction).

16. The reader might be concerned that some equilibria in the game without voting are no longer equilibria in the game with voting. However, we have shown that the winner of the contest is the same in all equilibria when voting is allowed; hence, the efficiency of the equilibria is indeed unaffected by allowing for voting at the end.

- In the case of quantity-restricted shares and votes $F_j = (\mathbf{R}_+ \times [0,1])^2$ is a set of $(p^{\mathrm{s}}, \overline{m}^{\mathrm{s}}; p^{\mathrm{v}}, \overline{m}^{\mathrm{v}})$ four-tuples (prices and corresponding quantity restrictions).
- In the case of contingent offers for shares, $F_j = \mathbf{R}_+^2$ is a pair of prices p^{s} and p^{sc} (non-contingent or contingent).
- In the case of contingent offers for shares and votes, $F_j = \mathbf{R}_+^4$ is a pair of pairs of prices, one pair corresponds to the contingent and non-contingent offers for shares and the other for votes.

First note that C is a non-empty correspondence. This follows from existence of equilibria in the shareholder subgame. Fix the offers, f_R, f_I. For each $\pi \in [0,1]$, define the set of tendering outcomes $M(\pi)$ that are optimal for the shareholders when they expect R to win with probability π. (That is, given π, if $m_k^h > 0$, then tendering h to k maximizes the shareholder's utility out of the available options, and if $\sum_{k,h} m_k^h < 1$, then not tendering must be optimal.) Clearly, this set of tendering outcomes is non-empty, convex valued and the correspondence $M(\pi)$ is upper hemi-continuous. Recall that for each outcome $m \in M(\pi)$, the correspondence $\Pi(m)$ defines the set of π's that are consistent with m. (That is, if R's share of the votes at that outcome is strictly smaller than $1/2$ or strictly larger than $1/2$, then the resulting set is $\{0\}$ or $\{1\}$, respectively; if R's share of the votes is exactly $1/2$, then the resulting set is $[0,1]$.) So $\Pi(M(\pi))$ defines a non-empty, convex valued, upper hemi-continuous correspondence whose fixed point is an equilibrium value of π for the tendering subgame. This implies that the set of all equilibrium outcomes $(m,\pi)_{f_R, f_I}$ in the tendering subgame following (f_R, f_I) is non-empty, and obviously $C(f_R, f_I)$ is a non-empty subset.

Now consider a different type of game in which we, the analysts, choose a selection of C. That is, we choose a function c defined on $F_R \times F_I$ such that $c(f_R, f_I) \in C(f_R, f_I)$ and other than that the game is the same as the original game. We call this the new game, and the preceding version—where the shareholders get to choose any equilibrium outcome of the tendering subgame from C—the original game.

Claim 1 *Given an SPE of the original game, there is a selection c under which those strategies are an SPE of the new game, and conversely, given a selection c and an SPE equilibrium of the new game, we have an SPE of the original game.*[17]

Proof. Obvious. ‖

Remark 3. In the original game, there is no selection from U_i that is continuous. Equivalently, there is no selection c such that the new game is continuous. To see this consider, *e.g.* parameters satisfying $\min_i(\omega_i + b_i) > p_I^{\mathrm{s}} > p_R^{\mathrm{s}} > \max_i \omega_i$. Then the only outcome that is not Pareto dominated by a strict equilibrium outcome in the tendering subgame is for all shareholders to sell to I. Consider $p_R^{\mathrm{s}} > p_I^{\mathrm{s}} > \max_i \omega_i$, then all sell to R. So if we have a sequence converging to $p_I^{\mathrm{s}} = p_R^{\mathrm{s}}$, continuity must fail: whatever we think shareholders do, the game is not continuous.

Claim 2 *C and U are upper hemi-continuous.*

Proof. Obvious. ‖

17. Here and elsewhere in this section, the term SPE refers to any subgame perfect equilibrium not necessarily a robust one (which we refer to as an equilibrium throughout the paper).

Remark 4. Note that if the set C was defined to include only Pareto undominated equilibrium outcomes in the tendering subgame (rather than all those that are undominated by *strict* equilibria of the tendering subgame), then we would not obtain upper hemi-continuity. Indeed, consider a game with $\omega_R > \omega_I$ and a subgame after $p_R^{\mathrm{s}} = 0$, $p_R^{\mathrm{v}} < b_I$. Then I has no best reply. I would want to choose $p_I^{\mathrm{v}} = p_R^{\mathrm{v}}$ and sell to all but this will be Pareto dominated for the shareholders by an (non-strict) equilibrium in the subgame in which all sell their votes to R . If I chooses $p_I^{\mathrm{v}} = p_R^{\mathrm{v}} + \varepsilon$, then I gets $u_I^{\varepsilon} = b_I - p_R^{\mathrm{v}} - \varepsilon$, so I wants to choose $\varepsilon > 0$ as small as possible.

Now define another game, call it an extended game.[18] The extended game has three players. The incumbent and rival have the same strategy space, and a fictitious third player chooses an element of $\mathbf{R}^2$. The pay-offs are as follows. I gets whatever the third player chooses for him, R gets whatever the third player chooses for him, and the third player's utility function is constant at 1 if the vector of strategies are any element of $\{(f_R, f_I, U_R(f_R, f_I), U_I(f_R, f_I))\} \subset F_R \times F_I \times \mathbf{R}^2$ and is a continuous function that strictly decreases as the strategies move away from that set. The pay-offs for I and R are trivially continuous. The pay-offs for the third player are continuous if (and only if) both U_ks are upper hemi-continuous.

Claim 3 *An SPE of the extended game is an SPE of a new game (where we use the selection c given by the third player from the extended game) and conversely.*

Proof. Obvious. ‖

Claim 4 *(Hellwig et al., 1990) Given any sequence of finite grids of a continuous extensive form game and any sequence of SPE for the sequence of games, the limit of the path of those SPE is an SPE path of the limit game. (Take subsequences whenever necessary.) Moreover, there exists a sequence of SPE of the finite games converging to the SPE of the limit game.*

Proof. The first claim is Theorem 1 in Hellwig *et al.* (1990). The second claim follows from their discussion of lower hemi-continuity (p. 419). ‖

Our existence result now follows from the above arguments.

Proposition 1. *In each of the scenarios considered in this paper, there exists an SPE whose outcome is a limit of SPE outcomes in a sequence of discretized versions of the game converging to the original game.*

Proof. Take a sequence of finite-grid games G_n converging to the original game, and take any convergent sequence of outcomes e_n such that e_n is an SPE of G_n. Any such outcome e_n is also an SPE outcome of an extended version of G_n (by the construction above). Hence, the extended version of the limit game has an SPE and furthermore the sequence e_n converges to the outcome of that SPE (by Hellwig *et al.*, 1990). The SPE that supports that outcome in the extended version of the limit game is an SPE of the original game that has the same outcome (by the construction above). ‖

We conclude by claiming that (robust) equilibria exist. First we make a trivial observation that follows from the definition of robustness.

18. We thank Phil Reny for this idea.

Claim 5 *Fix a sequence of grids without ties, F_k^n, $k = R, I$, such that $F_k^n \to F_k$. If $(f_R^n, \sigma_I^n, \{(m^n, \pi^n)_{f_R, f_I} \in C(f_R, f_I)\colon f_R, f_I \in F_R^n \times F_I^n\})$ is a sequence of (robust) equilibria with $f_R^n \to f_R$, $\sigma_I^n \to \sigma_I$ (i.e. for all $f_R \in F_R$, there is a sequence $f_R^n \to f_R$ with $\sigma_I^n(f_R^n) \to \sigma_I(f_I)$), and $(m^n, \pi^n) \to (m, \pi)$ $\big($i.e. for all (f_R, f_I), there is a sequence $(f_R^n, f_I^n) \to (f_R, f_I)$ with $(m^n, \pi^n)_{f_R^n, f_I^n} \to (m, \pi)_{f_R, f_I}\big)$ and $(f_R, \sigma_I, \{(m, \pi)_{f_R, f_I}\colon f_R, f_I \in F_R \times F_I\})$ is an SPE, then $\big(f_R, \sigma_I, \big\{(m, \pi)_{f_R, f_I}\colon f_R, f_I \in F_R \times F_I\big\}\big)$ is a (robust) equilibrium.*

Proof. This is just a restatement of the definition of robust equilibrium. ‖

Proposition 2. *A robust equilibrium exists in all the games considered in this paper.*

Proof. Follows from Claims 4 and 5 and Proposition 1. ‖

Remark 5. Note that the set of (robust) equilibrium outcomes is contained in the set of outcomes of SPE that satisfy the tie-free part of the robustness definition and such that, for any offers (f_R, f_I), $(m, \pi)_{(f_R, f_I)} \in C(f_R, f_I)$. This is because, if an outcome $(m, \pi)_{(f_R, f_I,)}$ is not an element of $C(f_R, f_I)$ because it is Pareto dominated by a strict equilibrium, say $(\hat{m}, \hat{\pi})$ in the tendering subgame, then it will also fail robustness. To see this, recall that robustness requires $(f_R^\varepsilon, f_I^\varepsilon)$ close to (f_R, f_I) and $(m^\varepsilon, \pi^\varepsilon)$ an equilibrium in the subgame following $(f_R^\varepsilon, f_I^\varepsilon)$ such that $(m^\varepsilon, \pi^\varepsilon)$ is not dominated by any strict equilibrium in the subgame following $(f_R^\varepsilon, f_I^\varepsilon)$. But for ε small enough, $(\hat{m}, \hat{\pi})$ will be a strict equilibrium in the subgame following $(f_R^\varepsilon, f_I^\varepsilon)$ and it will Pareto dominate $(m^\varepsilon, \pi^\varepsilon)$. Thus, characterization results that hold for all SPE that satisfy this weaker condition hold automatically for all the (robust) equilibria.

9. CONCLUSION

This paper makes two types of contributions. First, it makes a methodological contribution to the analysis of takeover games with a continuum of shareholders. It suggests a way of dealing with the asymmetric strategies that are crucial for the analysis, develops arguments that facilitate characterization results without fully constructing the set of equilibria, and deals with the question of existence. This opens the way both to examine and fully understand the scope of old results and to generate new results. Second, the analysis obtains relatively sharp substantive insights and shows that earlier conclusions might be misleading. The practice of vote buying is detrimental to efficiency under all circumstances but is not necessarily detrimental to shareholder profits. Thus, previous conclusions about the efficiency of vote buying when contingent offers are allowed and about the optimality of one share–one vote for shareholders pay-offs are imprecise or incomplete.

APPENDIX

Proofs for Section 4.2

Theorem 2 *The efficient contender wins in equilibrium except in the following regions of the parameter space:*

1. *If $w_I + b_I > w_R + b_R$ and $b_R > 2b_I$, then R wins though I is the efficient contender.*
2. *If $w_I + b_I < w_R + b_R < w_I + 2b_I$ and $b_I > b_R$, then I wins though R is the efficient contender.*

The proof relies on Lemma A1 (which adapts Lemma 1 to this case) and Propositions A1 and A2, which are stated and proved below. The analysis is simplified by noting that without loss of generality (w.l.o.g.) I need only make an offer for either shares or votes, but not both together. If shareholders sell only votes or only shares, then of course the other offer is irrelevant. If shareholders are indifferent and buy both, then they must be indifferent so that $p_I^{\mathrm{s}} = \pi w_R + (1-\pi)w_I + p_I^{\mathrm{v}}$ and then I is indifferent as well. This argument does not apply to R as an offer that is not taken in equilibrium may still restrict I's replies.[19]

Lemma A1. *There is no equilibrium in which both contenders have a strictly positive probability of winning, i.e. there is no equilibrium with* $\pi \in (0,1)$.

Proof. Note that in any equilibrium with $\pi \in (0,1)$, contender R purchases half the votes (with or without the shares) and the shareholders are indifferent. As in the proof of Lemma 1, robustness implies that, in any equilibrium, it cannot be that some shareholders sell some shares to I and some to R because any tie-free offers near $(p_R^{\mathrm{s}}, p_I^{\mathrm{s}})$ will break the indifference and change the outcome discontinuously. The proof of Lemma 1 also shows that it cannot arise due to shareholder indifference between tendering shares to R and not tendering (note that the argument there applies since such indifference requires $p_I^{\mathrm{v}} = p_R^{\mathrm{v}} = 0$.) Therefore, $\pi \in (0,1)$ can arise only in two cases. (1) After $(p_R^{\mathrm{v}}, p_R^{\mathrm{s}}, p_I^{\mathrm{v}})$ such that $p_R^{\mathrm{s}} \geq \min w_k$, $p_I^{\mathrm{v}} \in (p_R^{\mathrm{s}} - \max w_k, p_R^{\mathrm{s}} - \min w_k)$, and $p_I^{\mathrm{v}} \geq p_R^{\mathrm{v}}$ and no one sells votes to R.[20] (2) After $(p_R^{\mathrm{v}}, p_R^{\mathrm{s}}, p_I^{\mathrm{s}})$ such that $p_I^{\mathrm{s}} \in (p_R^{\mathrm{v}} + \min w_k, p_R^{\mathrm{v}} + \max w_k)$ and $p_I^{\mathrm{s}} \geq p_R^{\mathrm{s}}$ and no one sells shares to R.[21] Outside the closure of these open intervals R or I wins with certainty since all shareholders prefer selling either to I or to R regardless of π. (At the end points of these intervals, we have $p_k^{\mathrm{s}} = p_j^{\mathrm{v}} + w_l$ for $j \neq k$ and $l = I$ or R, which precludes $\pi \in (0,1)$ as shareholders indifference requires $p_k^{\mathrm{s}} = p_j^{\mathrm{v}} + \pi w_R + (1-\pi)w_I$ and $w_I \neq w_R$.)

First, consider the tendering subgame after offers $p_R^{\mathrm{s}} \geq \min w_k$ and $p_I^{\mathrm{v}} \geq 0$ such that $p_I^{\mathrm{v}} \in (p_R^{\mathrm{s}} - \max w_k, p_R^{\mathrm{s}} - \min w_k)$.

Assume $w_I > w_R$, so that $p_I^{\mathrm{v}} \in (p_R^{\mathrm{s}} - w_I, p_R^{\mathrm{s}} - w_R)$. The Pareto undomination part of the robustness requirement then selects $\pi = 0$.

Assume $w_I < w_R$ so that $p_I^{\mathrm{v}} \in (p_R^{\mathrm{s}} - w_R, p_R^{\mathrm{s}} - w_I)$. Then $\pi \in (0,1)$ implies that

$$p_R^{\mathrm{s}} = \pi w_R + (1-\pi)w_I + p_I^{\mathrm{v}} \tag{A.1}$$

and so

$$\pi = \frac{p_R^{\mathrm{s}} - w_I - p_I^{\mathrm{v}}}{w_R - w_I}; \tag{A.2}$$

hence,

$$\begin{aligned} u_I &= (1-\pi)b_I - p_I^{\mathrm{v}}/2 \\ &= \frac{w_R - p_R^{\mathrm{s}} + p_I^{\mathrm{v}}}{w_R - w_I} b_I - p_I^{\mathrm{v}}/2. \end{aligned} \tag{A.3}$$

Note that u_I describes the profit at the purported asymmetric equilibrium. Moreover, for other $p_I^{\mathrm{v}} \in (p_R^{\mathrm{s}} - w_R, p_R^{\mathrm{s}} - w_I)$, this function continues to describe the pay-offs to I so long as $p_I^{\mathrm{v}} > p_R^{\mathrm{v}}$.

If $w_I + 2b_I > w_R$, then u_I is increasing in p_I^{v} so I has a profitable deviation from the purported equilibrium.

If $w_I + 2b_I < w_R$, then u_I is decreasing in p_I^{v}, and if $p_I^{\mathrm{v}} > p_R^{\mathrm{v}}$, then there is again a profitable deviation for I from the purported equilibrium.

Thus, the only possibility for $\pi \in (0,1)$ is that $w_I + 2b_I < w_R$ with $p_R^{\mathrm{s}} \leq w_R$ (since if $p_R^{\mathrm{s}} > w_R$, then $u_I < 0$ by equation (A.3)) and $p_I^{\mathrm{v}} = p_R^{\mathrm{v}}$ (and no one sells votes to R). But this is ruled out as follows.

R's pay-off at the purported equilibrium is

$$\begin{aligned} u_R &= \pi b_R + \frac{\pi w_R + (1-\pi)w_I - p_R^{\mathrm{s}}}{2} \\ &= \pi b_R - p_I^{\mathrm{v}}/2 \\ &= \frac{p_R^{\mathrm{s}} - w_I - p_I^{\mathrm{v}}}{w_R - w_I} b_R - p_I^{\mathrm{v}}/2, \end{aligned} \tag{A.4}$$

19. For example, if $p_R^{\mathrm{v}} + \pi w_R + (1-\pi)w_I = p_I^{\mathrm{v}} + \pi w_R + (1-\pi)w_I = p_R^{\mathrm{s}}$, it may be that no shareholders buy votes from R and I fails to lower p_I^{v} as that would result in no one selling votes to I. But if R were to lower p_R^{v}, then I could lower p_I^{v} and not lose all votes.

20. No one sells votes to R because in any tie-free offers, either $p_I^{\mathrm{v}} > p_R^{\mathrm{v}}$ and no one sells votes to R or $p_I^{\mathrm{v}} < p_R^{\mathrm{v}}$ and then no one would sell votes or shares to I, and in both events, by the tie-free part of the robustness requirement, π would not be interior.

21. See footnote 20.

which is increasing in $p^{\rm s}_R$ and decreasing in $p^{\rm v}_I$. If R deviates to $p^{\rm s}_R = w_R$ and $p^{\rm v}_R = 0$, then I will not respond with $p^{\rm s}_I \geq p^{\rm s}_R$ (since if the last inequality is strict, then $u_I = w_I - p^{\rm s}_I < w_I - w_R < 0$, and if it is an equality, then by the tie-free part of the robustness requirement either I buys from all and also $u_I = w_I - p^{\rm s}_I = w_I - w_R < 0$ or R buys from all and $u_I = 0$), and as established above in this case, u_I is decreasing in $p^{\rm v}_I$, so I's best response in terms of $p^{\rm v}_I$ is $p^{\rm v}_I = 0$. Therefore, the deviation to $p^{\rm s}_R = w_R$ and $p^{\rm v}_R = 0$ increases u_R, so R has a profitable deviation unless $p^{\rm s}_R = w_R$ and $p^{\rm v}_R = 0$. But then, as noted, I's best reply is $p^{\rm v}_I = 0$ whereupon $\pi = 1$. This establishes that in the subgame following an offer $p^{\rm s}_R \geq \min w_k$, there is no equilibrium with $\pi \in (0,1)$.

Second, consider the equilibria in the subgame following $(p^{\rm v}_R, p^{\rm s}_I)$ such that $p^{\rm s}_I$ is in the interval $(p^{\rm v}_R + \min w_k, p^{\rm v}_R + \max w_k)$.

If $w_R > w_I$, then there are multiple shareholder equilibria, but again the Pareto undomination part of the robustness requirement selects the equilibrium where all sell to R so $\pi = 1$.

If $w_I > w_R$, then shareholder indifference implies that

$$p^{\rm v}_R + \pi w_R + (1-\pi) w_I = p^{\rm s}_I \tag{A.5}$$

and hence

$$\pi = \frac{p^{\rm v}_R + w_I - p^{\rm s}_I}{w_I - w_R}. \tag{A.6}$$

$$\begin{aligned} u_I &= (1-\pi) b_I + \frac{\pi w_R + (1-\pi) w_I - p^{\rm s}_I}{2} \\ &= \frac{p^{\rm s}_I - w_R - p^{\rm v}_R}{w_I - w_R} b_I - \frac{p^{\rm v}_R}{2}, \end{aligned} \tag{A.7}$$

which is linear and increasing in $p^{\rm s}_I$ over $[w_R + p^{\rm v}_R, w_I + p^{\rm v}_R]$. Therefore, $\max u_I$ is achieved at $p^{\rm s}_I = w_I + p^{\rm v}_R$, where $\pi = 0$. Thus, if $w_I > w_R$, then $\pi \notin (0,1)$.

It follows that for all parameter configurations, $\pi \in (0,1)$ does not arise on the equilibrium path. ‖

Proposition A1. *If (i) $w_R + b_R > w_I + 2b_I$ or (ii) $b_R > 2b_I$ or (iii) $w_R + b_R > w_I + b_I$ and $b_R > b_I$, then I may not win in equilibrium.*

Proof.

(A) If $w_R + b_R > w_I + 2b_I$ and $w_R > w_I$, then R can start with $p^{\rm s}_R$ in the interval $(\max\{w_I + 2b_I, w_R\}, w_R + b_R)$ and win profitably. To see this, observe first that it would not be profitable for I to respond with $p^{\rm s}_I \geq p^{\rm s}_R > w_I + 2b_I$. Suppose next that I responds with $p^{\rm v}_I$. Clearly $p^{\rm v}_I < p^{\rm s}_R - w_R$ leads to $\pi = 1$ (this inequality implies that selling shares to R is better for shareholders than selling votes to I) and $p^{\rm v}_I > p^{\rm s}_R - w_I$ leads to losses for I (since then $p^{\rm v}_I > 2b_I$ and the best I can do is buy half the votes and obtain control with probability 1). For $p^{\rm v}_I \in [p^{\rm s}_R - w_R, p^{\rm s}_R - w_I]$, equations (A.1) and (A.2) hold, so $u_I = \frac{w_R + p^{\rm v}_I - p^{\rm s}_R}{w_R - w_I} b_I - \frac{p^{\rm v}_I}{2}$, and, over this range, u_I is maximized either at $p^{\rm v}_I = p^{\rm s}_R - w_R > 0$ which implies $\pi = 1$ (because if $\pi < 1$, then tendering votes to I yields less than tendering shares to R, so cannot happen in equilibrium) or at $p^{\rm v}_I = p^{\rm s}_R - w_I$ which implies $u_I = b_I - \frac{p^{\rm v}_I}{2} < 0$.

(B) If $b_R > 2b_I$ and $w_R < w_I$ ($w_R > w_I$ is covered by the preceding case), then R can start with $p^{\rm v}_R > 2b_I$ and win profitably. To see this, observe first that it would not be profitable for I to respond with $p^{\rm v}_I \geq p^{\rm v}_R$. Suppose that I responds with $p^{\rm s}_I$. Clearly, $p^{\rm s}_I < w_R + p^{\rm v}_R$ results in $\pi = 1$ and $p^{\rm s}_I > w_I + p^{\rm v}_R$ leads to losses for I. Otherwise equation (A.5) holds, $w_R \leq p^{\rm s}_I - p^{\rm v}_R \leq w_I$, π is given by equation (A.6) and $u_I = \frac{p^{\rm s}_I - w_R - p^{\rm v}_R}{w_I - w_R} b_I - \frac{p^{\rm v}_R}{2}$. Hence, $p^{\rm v}_R > 2b_I$ implies that $u_I < 0$, which means that I's best response is to let R win.

(C) Suppose $w_I + b_I < w_R + b_R$ and $b_I < b_R$. First we argue that if $w_I > w_R$, then it cannot be that $\pi = 0$. If R offers $p^{\rm s}_R \in (w_I + b_I, w_R + b_R)$, then I has no profitable counter-offer and R has profits. To see that I has no profitable counter-offer, first note that $p^{\rm s}_I \geq p^{\rm s}_R > w_I + b_I$, then all tender to I, so this cannot lead to gains for I. Next, if $p^{\rm v}_I < p^{\rm s}_R - w_I$, then $\pi = 1$. If $p^{\rm v}_I > p^{\rm s}_R - w_I$, then by the Pareto undomination part of the robustness requirement, all shareholders tender votes to I and $u_I = b_I - p^{\rm v}_I < b_I + w_I - p^{\rm s}_R < 0$. If $p^{\rm v}_I = p^{\rm s}_R - w_I$ and not everyone sells to I and I wins, then I may have a profit. But this is ruled out by the tie-free part of the robustness requirement.

If $w_R > w_I$, then it cannot be that $\pi = 0$. If R offers $p^{\rm v}_R \in (b_I, b_R)$, then I has no profitable counter-offer and R has profits. To see that, I has no profitable counter-offer, first note that $p^{\rm v}_I > p^{\rm v}_R$ can only lead to losses. If $p_I^{\mathrm{s}} < \omega_R + p_R^{\mathrm{v}}$, then (due to the Pareto undomination part of the robustness requirement) I loses. If $p_I^{\mathrm{s}} > \omega_R + p_R^{\mathrm{v}}$, then all shareholders sell to I and I has losses. Finally, if $p_I^{\mathrm{s}} = \omega_R + p_R^{\mathrm{v}}$ and not everyone sells to I and I wins, then I may have a profit. But this is ruled out by tie-free part of the robustness requirement. (A) and (B) together cover cases (i) and (ii), while (C) covers (iii). ‖

Proposition A2. *If (i) $\omega_R + b_R < \omega_I + b_I$ and $b_R < 2b_I$ or (ii) $b_R < b_I$ and $\omega_R + b_R < \omega_I + 2b_I$, then it cannot be that R wins.*

Proof. First consider the case $\omega_R + b_R < \omega_I + b_I$ and $b_R < 2b_I$. If $\pi = 1$, then either $p_R^{\mathrm{s}} \geq \omega_I + b_I$ or $p_R^{\mathrm{v}} \geq b_I$. (Otherwise I has a profitable deviation.) But if $p_R^{\mathrm{s}} \geq \omega_I + b_I$, then with $\pi = 1$ all shareholders tender shares to R, so R has a loss, since $\omega_R + b_R < \omega_I + b_I$. If $p_R^{\mathrm{v}} \geq b_I$, then there are two possibilities. If $\omega_R > \omega_I$, in which case $b_I > b_R$ (since $\omega_R + b_R < \omega_I + b_I$), then $p_R^{\mathrm{v}} > b_R$ and with $\pi = 1$ all tender votes to R and that implies again that R has a loss. If $\omega_R < \omega_I$, then I can set p_I^{s} just below $\omega_I + p_R^{\mathrm{v}}$ and win profitably with (just above) half the shareholders selling to I which is profitable for I while R has a loss. This proves (i).

If (ii) holds (but not (i)), then $b_R < b_I$ and $\omega_I + b_I < \omega_R + b_R < \omega_I + 2b_I$, so $\omega_R > \omega_I$, and if $\pi = 1$, then either $p_R^{\mathrm{v}} \geq b_I > b_R$ and all tender votes to R and R has losses, or $p_R^{\mathrm{s}} > \omega_R + b_R$ and all tender shares to R and R has losses, or $p_R^{\mathrm{s}} \leq \omega_R + b_R$. But then if I offers $p_I^{\mathrm{v}} = p_R^{\mathrm{s}} - \omega_I - \varepsilon < 2b_I$, the only equilibrium in the tendering subgame is asymmetric with $\pi \approx 0$ (since if all tender votes to I, it is better to tender shares to R [$p_R^{\mathrm{s}} > p_I^{\mathrm{v}} + \omega_I$] and if all tender shares to R, it is better to tender one's vote to I [as $p_I^{\mathrm{v}} + \omega_R > p_R^{\mathrm{s}}$], so the equilibrium in the tendering subgame must be asymmetric with $p_R^{\omega} = p_I^{\mathrm{v}} + \pi\omega_R + (1-\pi)\omega_I$ so that $p_I^{\mathrm{v}} \approx p_R^{\mathrm{s}} - \omega_I \Rightarrow \pi \approx 0$) and this is profitable to I. ‖

Proof of Theorem 2. To see how the result follows from Lemma A1 and Propositions A1 and A2, we partition the parameter space as follows. Cases 2 and 4 below are those that correspond to Cases 1 and 2 in the statement of the theorem.

1. $\omega_R + b_R < \omega_I + b_I$ and $b_R < 2b_I$ where I wins.
2. $\omega_R + b_R < \omega_I + b_I$ and $b_R > 2b_I$ where R wins.
3. $\omega_R + b_R > \omega_I + b_I$ and $b_I < b_R$ where R wins.
4. $\omega_I + 2b_I > \omega_R + b_R > \omega_I + b_I$ and $b_I > b_R$ where I wins.
5. $\omega_R + b_R > \omega_I + 2b_I$ $(> \omega_I + b_I)$ and $b_I > b_R$ where R wins.

By Lemma A1 and the existence result, in all equilibria either R wins or I wins with probability 1. Then Proposition A1 part (i) implies 5, part (ii) implies 2 (and part of 3), and part (iii) implies part 3. Proposition A2 part (i) implies 1, part (ii) implies 4 (and part of 1). ‖

Proofs for Section 5.1

Theorem 3 *In all equilibria, the contender with the higher value of $\omega_j + 2b_j$ wins.*

Proof. First we observe that, w.l.o.g. we can restrict attention to I's offers $(p_I^{\mathrm{s}}, \overline{m}_I^{\mathrm{s}})$ with $\overline{m}_I^{\mathrm{s}} = 1/2$. To see this, observe that, for given levels of m_I^{s} and π, any offer $(p_I^{\mathrm{s}}, \overline{m}_I^{\mathrm{s}})$ is equivalent for shareholders to $(\hat{p}_I^{\mathrm{s}}, 1/2)$, where $\hat{p}_I^{\mathrm{s}}$ satisfies

$$(\hat{p}_I^{\mathrm{s}} - [\pi\omega_R + (1-\pi)\omega_I])\min\left\{\frac{1}{2m_I^{\mathrm{s}}}, 1\right\} = (p_I^{\mathrm{s}} - [\pi\omega_R + (1-\pi)\omega_I])\min\left\{\frac{\overline{m}_I^{\mathrm{s}}}{m_I^{\mathrm{s}}}, 1\right\}. \tag{A.8}$$

Therefore, there exists an equilibrium in the tendering subgame following $(\hat{p}_I^{\mathrm{s}}, 1/2)$ with the same m_I^{s} and π. Let u_I denote I's profit with $(p_I^{\mathrm{s}}, \overline{m}_I^{\mathrm{s}})$

$$u_I = (1-\pi)b_I + \min(\overline{m}_I^{\mathrm{s}}, m_I^{\mathrm{s}})[\pi\omega_R + (1-\pi)\omega_I - p_I^{\mathrm{s}}]$$

and let $\hat{u}_I$ denote I's profit with $(\hat{p}_I^{\mathrm{s}}, 1/2)$ and the same π

$$\hat{u}_I = (1-\pi)b_I + \min(1/2, m_I^{\mathrm{s}})[\pi\omega_R + (1-\pi)\omega_I - \hat{p}_I^{\mathrm{s}}].$$

From $\min(x, m_I^{\mathrm{s}}) = m_I^{\mathrm{s}}\min[(x/m_I^{\mathrm{s}}), 1]$ and equation (A.8), it follows that $u_I = \hat{u}_I$. Therefore, there exists an equilibrium in the tendering subgame following $(\hat{p}_I^{\mathrm{s}}, 1/2)$ at which I gets the same profit as in the equilibrium of the tendering subgame following $(p_I^{\mathrm{s}}, \overline{m}_I^{\mathrm{s}})$.

If $\pi \in (0,1)$ arises at equilibrium, it must be that $m_R^{\mathrm{s}} = 1/2$, $\overline{m}_R^{\mathrm{s}} \geq 1/2$, and $m_I^{\mathrm{s}} \leq 1/2$. It cannot be that $m_I^{\mathrm{s}} = 1/2$ and that I is oversubscribed because then fewer than $1/2$ tender to R and I wins. If $\overline{m}_R^{\mathrm{s}} = 1/2$ and R is oversubscribed, then R wins (by our specification above—see Remark 1). But then it cannot be that shareholders are selling shares to both I and R since this is ruled out by the tie-free part of the robustness requirement. Thus, shareholders must be indifferent between selling to R and not tendering at all, implying that $p_R^s = \pi w_R + (1-\pi)w_I$. Let u_j denote the profit of $j = I, R$ in the putative equilibrium with $\pi \in (0,1)$.

$$u_I = (1-\pi)b_I = (1-\pi)b_I, \tag{A.9}$$

$$u_R = \frac{1}{2}[-p_R^s + \pi w_R + (1-\pi)w_I] + \pi b_R = \pi b_R. \tag{A.10}$$

Consider the following two configurations of parameters.

1. Suppose $w_I + 2b_I > w_R + 2b_R$ and that $\pi > 0$. It may not be that $\pi = 1$ since R's profitability implies $p_R^s \leq w_R + 2b_R$, but then I can win profitably with $(p_I^s, \overline{m}_I^s) = (\max\{p_R^s, w_R\}, 1/2)$. Thus, $\pi < 1$ in any equilibrium. Suppose then that $\pi \in (0,1)$, so that equations (A.9) and (A.10) hold. Consider a deviation by I to the offer $(p_I^s, \overline{m}_I^s) = (p_R^s + \varepsilon, 1/2)$, where ε is positive and small, say $\varepsilon < \pi 2b_R$. Contender I will end up buying from a mass $\theta \leq 0.5$ of the shareholders and win (since either $m_I^s > 1/2$, and I wins, or $m_I^s \leq 1/2$, which implies that nobody would tender to R since tendering to I is more profitable). Let $\hat{u}_I$ denote I's profit following this deviation:

$$\begin{aligned}\hat{u}_I &= \theta(-p_R^s - \varepsilon + w_I) + b_I \\ &\geq \theta[-p_R^s - \varepsilon + \pi(w_R + 2b_R) + (1-\pi)(w_I + 2b_I)] + (1-2\theta)b_I \\ &= \theta[-\varepsilon + \pi 2b_R + (1-\pi)2b_I] + (1-2\theta)b_I > (1-\pi)b_I = u_I,\end{aligned}$$

where the first inequality follows from the assumption $w_I + 2b_I > w_R + 2b_R$. Thus, I can deviate profitably from the putative equilibrium with $\pi \in (0,1)$. Together with the previous observation that $\pi < 1$, we have that there is no equilibrium with $\pi > 0$. Combining this with the result on existence, we conclude that with these parameters $\pi = 0$.
2. Suppose $w_I + 2b_I < w_R + 2b_R$ and that $\pi < 1$. It may not be the case that $\pi = 0$ since $p_R^s > \max\{w_I + 2b_I, w_R\}$ and $\overline{m}_R^s = 1/2$ would guarantee a profitable win for R, which I can defeat only at a loss. Therefore, $\pi \in (0,1)$ and again $p_I^s \leq p_R^s = \pi w_R + (1-\pi)w_I$ and equations (A.9) and (A.10) hold. Since it is an equilibrium, I cannot profitably outbid R with $(p_I^s, \overline{m}_I^s) = (p_R^s + \varepsilon, 1/2)$. That is,

$$u_I \geq b_I + (w_I - p_R^s)/2.$$

Since $p_R^s = \pi w_R + (1-\pi)w_I$, this implies that

$$u_I \geq b_I + (w_I - [\pi w_R + (1-\pi)w_I])/2 = (1-\pi)b_I + \pi(w_I + 2b_I - w_R)/2.$$

If $w_I + 2b_I > w_R$, it follows that $u_I > (1-\pi)b_I$ in contradiction to equation (A.9). If $w_I + 2b_I \leq w_R$, then $\pi \in (0,1)$ may not arise in equilibrium since $p_R^{s\prime} > w_R$ would guarantee R a win with profit $b_R + w_R - p_R^{s\prime}$. But, for $\hat{p}_R^s$ sufficiently close to w_R, $b_R + w_R - \hat{p}_R^s > \pi b_R \geq u_R$ in contradiction to equilibrium. Therefore, $\pi \in (0,1)$ cannot arise in equilibrium. Thus, there is no equilibrium with $\pi < 1$. Combining this with the result on existence we conclude that with these parameters $\pi = 1$. ‖

Proofs for Section 5.2

Theorem 4 *The identity of the winner is the same as in Theorem 3 except for parameter configurations satisfying $w_I + 2b_I > w_R + 2b_R$ and $b_R > b_I$. For these configurations, I is the efficient contestant and would be the winner in the absence of vote trading, but R wins when vote trading is allowed.*

Proof. The proof follows from the subsequent characterization of equilibrium outcomes and existence. By Lemma A2 and existence, $\pi \in \{0,1\}$. Propositions A3 and A4 preclude either $\pi = 0$ or $\pi = 1$ for all possible configurations of the parameters. ‖

Before proving that in equilibrium $\pi \notin (0,1)$, it is useful to establish that it suffices to restrict attention only to a subset of the possible offers, specifically to I making an offer $(p_I^s, 1/2; 0, 0)$ or $(0, 0; p_I^v, 1/2)$ and to R making an offer $(p_R^s, \overline{m}_R^s; p_R^v, \overline{m}_R^v)$ with $\overline{m}_R^s \geq 1/2$ and $\overline{m}_R^v \geq 1/2$. The next two claims formalize this result.

Claim A1 *For any $\pi \in (0,1)$ that arises in some tendering subgame following some f_R, f_I, there exists an equilibrium in the subgame following f_R in which I's offer is $(p_I^{\mathrm{s}}, 1/2; 0, 0)$ or $(0, 0; p_I^{\mathrm{v}}, 1/2)$ and the subsequent tendering subgame (following f_R and I's offer of $(p_I^{\mathrm{s}}, 1/2; 0, 0)$ or $(0, 0; p_I^{\mathrm{v}}, 1/2)$) has the same π. Moreover, if the original equilibrium in the tendering subgame is not Pareto dominated by any strict equilibrium in the tendering subgame, then neither is the equilibrium following f_R and I's offer of $(p_I^{\mathrm{s}}, 1/2; 0, 0)$ or $(0, 0; p_I^{\mathrm{v}}, 1/2)$ that has the same π.*

Proof. Suppose that I's offer in the original equilibrium is $(p_I^{\mathrm{s}}, \overline{m}_I^{\mathrm{s}}; p_I^{\mathrm{v}}, \overline{m}_I^{\mathrm{v}})$. If shareholders tender to I only shares (*i.e.* $m_I^{\mathrm{s}} > 0$ and $m_I^{\mathrm{v}} = 0$), this offer is equivalent to $(p_I^{\mathrm{s}}, \overline{m}_I^{\mathrm{s}}; 0, 0)$. For the shareholders, this is obviously equivalent to $(p_I^{\mathrm{s}\prime}, 1/2; 0, 0)$, where $p_I^{\mathrm{s}\prime}$ satisfies

$$(p_I^{\mathrm{s}\prime} - [\pi w_R + (1-\pi) w_I]) \min[(1/(2m_I^{\mathrm{s}})), 1] = (p_I^{\mathrm{s}} - [\pi w_R + (1-\pi) w_I]) \min[(\overline{m}_I^{\mathrm{s}}/m_I^{\mathrm{s}}), 1].$$

I's profit with $(p_I^{\mathrm{s}}, \overline{m}_I^{\mathrm{s}}; 0, 0)$ is

$$(1-\pi) b_I + \min(\overline{m}_I^{\mathrm{s}}, m_I^{\mathrm{s}})[\pi w_R + (1-\pi) w_I - p_I^{\mathrm{s}}].$$

Since π remains the same with $(p_I^{\mathrm{s}\prime}, 1/2; 0, 0)$, I's profit with $(p_I^{\mathrm{s}\prime}, 1/2; 0, 0)$ is

$$(1-\pi) b_I + \min(1/2, m_I^{\mathrm{s}})[\pi w_R + (1-\pi) w_I - p_I^{\mathrm{s}\prime}].$$

Since $\min(\overline{m}^{\mathrm{s}}, m_I^{\mathrm{s}}) = m_I^{\mathrm{s}} \min[(\overline{m}^{\mathrm{s}}/m_I^{\mathrm{s}}), 1]$, it follows that $(p_I^{\mathrm{s}}, \overline{m}_I^{\mathrm{s}}; 0, 0)$ and $(p_I^{\mathrm{s}\prime}, 1/2; 0, 0)$ are equivalent for I as well.

An analogous argument would establish that if shareholders tender to I only votes (*i.e.* $m_I^{\mathrm{s}} = 0$ and $m_I^{\mathrm{v}} > 0$), there is an equivalent offer $(0, 0; p_I^{\mathrm{v}\prime}, 1/2)$.

Suppose therefore that shareholders tender to I both votes and shares (*i.e.* $m_I^{\mathrm{s}} > 0$ and $m_I^{\mathrm{v}} > 0$). This implies that they are indifferent between these two options. That is, $\pi w_R + (1-\pi) w_I + \min\{\overline{m}_I^{\mathrm{v}}/m_I^{\mathrm{v}}, 1\} p_I^{\mathrm{v}} = \min\{\overline{m}_I^{\mathrm{s}}/m_I^{\mathrm{s}}, 1\} p_I^{\mathrm{s}} + (1 - \min\{\overline{m}_I^{\mathrm{s}}/m_I^{\mathrm{s}}, 1\}) \times [\pi w_R + (1-\pi) w_I]$.

Clearly, the offer $(p_I^{\mathrm{s}}, \overline{m}_I^{\mathrm{s}\prime}; 0, 0)$ such that $\overline{m}_I^{\mathrm{s}\prime} = \min\{\overline{m}_I^{\mathrm{s}}(m_I^{\mathrm{s}} + m_I^{\mathrm{v}})/m_I^{\mathrm{s}}, 1\}$ is equivalent for the shareholders if $m_I^{\mathrm{s}} + m_I^{\mathrm{v}}$ tender to it. To see that it is also equivalent for I, observe that I's profit with $(p_I^{\mathrm{s}}, \overline{m}_I^{\mathrm{s}\prime}; 0, 0)$ equals

$$\begin{aligned}
&(1-\pi) b_I + \min\{\overline{m}_I^{\mathrm{s}\prime}, m_I^{\mathrm{s}} + m_I^{\mathrm{v}}\}[\pi w_R + (1-\pi) w_I - p_I^{\mathrm{s}}] \\
&= (1-\pi) b_I + (m_I^{\mathrm{s}} + m_I^{\mathrm{v}}) \min\{\overline{m}_I^{\mathrm{s}\prime}/(m_I^{\mathrm{s}} + m_I^{\mathrm{v}}), 1\}[\pi w_R + (1-\pi) w_I - p_I^{\mathrm{s}}] \\
&= (1-\pi) b_I + (m_I^{\mathrm{s}} + m_I^{\mathrm{v}}) \min[\min\{\overline{m}_I^{\mathrm{s}}/m_I^{\mathrm{s}}, 1/(m_I^{\mathrm{s}} + m_I^{\mathrm{v}})\}, 1][\pi w_R + (1-\pi) w_I - p_I^{\mathrm{s}}] \\
&= (1-\pi) b_I + m_I^{\mathrm{s}} \min\{\overline{m}_I^{\mathrm{s}}/m_I^{\mathrm{s}}, 1\}[\pi w_R + (1-\pi) w_I - p_I^{\mathrm{s}}] \\
&\quad + m_I^{\mathrm{v}} \min\{\overline{m}_I^{\mathrm{s}}/m_I^{\mathrm{s}}, 1\}[\pi w_R + (1-\pi) w_I - p_I^{\mathrm{s}}] \\
&= (1-\pi) b_I + m_I^{\mathrm{s}} \min\{\overline{m}_I^{\mathrm{s}}/m_I^{\mathrm{s}}, 1\}[\pi w_R + (1-\pi) w_I - p_I^{\mathrm{s}}] - m_I^{\mathrm{v}} \min\{\overline{m}_I^{\mathrm{v}}/m_I^{\mathrm{v}}, 1\} p_I^{\mathrm{v}} \\
&= (1-\pi) b_I + \min\{\overline{m}_I^{\mathrm{s}}, m_I^{\mathrm{s}}\}[\pi w_R + (1-\pi) w_I - p_I^{\mathrm{s}}] - \min\{\overline{m}_I^{\mathrm{v}}, m_I^{\mathrm{v}}\} p_I^{\mathrm{v}},
\end{aligned}$$

which equals I's profit with $(p_I^{\mathrm{s}}, \overline{m}_I^{\mathrm{s}}; p_I^{\mathrm{v}}, \overline{m}_I^{\mathrm{v}})$.

The second equality follows from the definition of $\overline{m}_I^{\mathrm{s}\prime}$, the third from $1/(m_I^{\mathrm{s}} + m_I^{\mathrm{v}}) \geq 1$, and the fourth from the shareholders' indifference.

Finally, it follows from the previous argument that $(p_I^{\mathrm{s}}, \overline{m}_I^{\mathrm{s}\prime}; 0, 0)$ is equivalent to $(p_I^{\mathrm{s}\prime}, 1/2; 0, 0)$.

It is straightforward to verify that the equilibrium in a tendering subgame that is constructed in this proof is not Pareto dominated by any strict equilibrium in the subgame if the original equilibrium in the tendering subgame was not Pareto dominated. ||

Claim A2 *For any $\pi \in (0,1)$ that arises in some tendering subgame following f_R, f_I, there exists an equilibrium in the tendering subgame following an offer by R, $(p_R^{\mathrm{s}}, \overline{m}_R^{\mathrm{s}}; p_R^{\mathrm{v}}, \overline{m}_R^{\mathrm{v}})$, that satisfies $\overline{m}_R^{\mathrm{s}} \geq 1/2$ or $\overline{m}_R^{\mathrm{v}} \geq 1/2$ and which has the same $\pi \in (0,1)$. Moreover, if the original equilibrium in the tendering subgame is not Pareto dominated by a strict equilibrium in the tendering subgame, then neither is the equilibrium of the tendering subgame that has the same π and follows the aforementioned restricted offers.*

Proof. Consider the case $\overline{m}_R^{\text{s}} < 1/2$ and $\overline{m}_R^{\text{v}} < 1/2$. It has to be that $\overline{m}_R^{\text{s}} + \overline{m}_R^{\text{v}} \geq 1/2$ since otherwise $\pi = 0$. Since $\pi \in (0,1)$, at least one of R's offers is not oversubscribed, for otherwise R would win. If offer p_R^{s} is not oversubscribed, then the offer $(p_R^{\text{s}}, 1/2; p_R^{\text{v}}, \overline{m}_R^{\text{v}})$ when coupled with the same response by I would leave the existing shareholders' tendering decisions optimal, hence would yield the same π and the same pay-offs for R and I. And if I has a better response against $(p_R^{\text{s}}, 1/2; p_R^{\text{v}}, \overline{m}_R^{\text{v}})$ than its original response, then this response would be also better against the original offer by R. An analogous argument can be made if it is p_R^{v} that is not oversubscribed, in which case the offer $(p_R^{\text{s}}, \overline{m}_R^{\text{s}}; p_R^{\text{v}}, 1/2)$ would achieve the same result against I's response.

We also need to argue why this construction does not violate the Pareto undomination part of the robustness requirement. If $\omega_R > p_R^{\text{s}}$, then the only equilibrium in the tendering subgame has $1/2$ selling to R; this is unchanged. If $\omega_R \leq p_R^{\text{s}}$, then if all sell to R they get $\omega_R/2 + p_R^{\text{s}}/2 \leq p_R^{\text{s}}$ which they get in the constructed equilibrium of the tendering subgame. ‖

Lemma A2. *There is no equilibrium in which both R and I have a strictly positive probability of winning, i.e. there is no equilibrium with $\pi \in (0,1)$.*

Proof. Suppose $\pi \in (0,1)$. This implies that R ends up acquiring exactly half votes (with or without shares) and that shareholders are indifferent between tendering to R and the alternative of tendering to I or keeping their shares. That is, $\min\{m_R^{\text{s}}, \overline{m}_R^{\text{s}}\} + \min\{m_R^{\text{v}}, \overline{m}_R^{\text{v}}\} = 1/2$. By the preceding claim at least one of R's offers is not restricted to quantity below $1/2$. That offer is not oversubscribed since if it were R would win. Thus, there must be indifference between that offer and the same alternative as there was in the second sentence of this paragraph.

Given these observations, the proof mimics that of Lemma A1 essentially verbatim. ‖

Proposition A3. *If $\omega_R + 2b_R > \omega_I + 2b_I$, or $b_R > b_I$, then I cannot win.*

Proof. If $\omega_R + 2b_R > \omega_I + 2b_I$, or $b_R > b_I$, it may not be that $\pi = 0$, since in the former case R can start with $(p_R^{\text{s}}, 1/2; 0, 0)$ such that $p_R^{\text{s}} \in (\omega_I + 2b_I, \omega_R + 2b_R)$ and in the latter case with $(0, 0; p_R^{\text{v}}, 1/2)$ such that $p_R^{\text{v}} > 2b_I$ and win profitably in both cases. ‖

Proposition A4. *If $\omega_R + 2b_R < \omega_I + 2b_I$ and $b_R < b_I$, then R cannot win.*

Proof. If R wins with probability 1, then either $p_R^{\text{s}} \geq \omega_I + 2b_I$ and $\overline{m}_R^{\text{s}} \geq 1/2$ or $p_R^{\text{v}} \geq 2b_I$ and $\overline{m}_R^{\text{v}} \geq 1/2$. In both cases, R has losses, so there is no such equilibrium. ‖

Proofs for Section 6.1

The following lemma narrows down the set of scenarios that have to be considered.

Lemma A3. *Given any robust equilibrium with outcome π, there is a robust equilibrium with outcome π when we restrict attention to the case where I makes only non-contingent offers and R does not make both types of offers, only one.*

Proof. We first argue that w.l.o.g. attention can be restricted to the case where I makes only non-contingent offers. Consider then the case in which I makes a contingent offer p_I^{sc}. In an asymmetric equilibrium of the tendering subgame, the shareholders would be indifferent either between tendering to R and to I or between tendering to R and just holding on to the shares. In the former case, the pay-off to a shareholder from tendering to I would be $(1-\pi)p_I^{\text{sc}} + \pi\omega_R$ and the pay-off to I would be $(1-\pi)b_I + \theta(1-\pi)(\omega_I - p_I^{\text{sc}})$, where $\theta \in [0, 1/2]$ is the fraction of shares tendered to I. It follows that, if I offers instead the non-contingent price $p_I^{\text{s}\prime} = (1-\pi)p_I^{\text{sc}} + \pi\omega_R$, the above outcome will continue to be an equilibrium of the tendering subgame. That is, the probability of R's win will continue to be π, a fraction θ will tender to I and those tendering to I and those who do not will receive the same pay-off. I's pay-off will be $(1-\pi)b_I + \theta[\pi\omega_R + (1-\pi)\omega_I - p_I^{\text{s}\prime}] = (1-\pi)b_I + \theta(1-\pi)(\omega_I - p_I^{\text{sc}})$ just as before. Thus, in an asymmetric equilibrium, w.l.o.g., we may assume that I is confined to making only non-contingent offers. So it is enough to examine contingent offers only by R.

The Pareto dominance part of the refinement might rule out an equilibrium with $\pi \in (0,1)$ under p_I^{sc} but not for $p_I^{\text{s}} = (1-\pi)p_I^{\text{sc}} + \pi\omega_R$. However, this does not affect the argument just given since whenever the Pareto dominance part of the refinement would rule out an equilibrium with $\pi \in (0,1)$ for $p_I^{\text{s}} = (1-\pi)p_I^{\text{sc}} + \pi\omega_R$, it would also rule it out for p_I^{sc}. The constructed equilibrium will satisfy the tie-free requirement as well since ties were not used in the construction, so if one happens to be created nearby actions will be tie-free and have approximately the same π.

When $\pi \in \{0, 1\}$, it is obvious that I can be confined to non-contingent offers w.l.o.g.—if without being confined I loses, then I continues to lose with a restricted strategy space; if without being confined I wins with probability 1, then the contingent offer is equivalent to a non-contingent offer. Clearly, in these cases the new strategies constitute a (robust) equilibrium.

Now we argue that w.l.o.g. attention can be restricted to the case where R does not make both contingent and non-contingent offers, just one of the two. If $\pi \in (0, 1)$, shareholders must be indifferent between R's contingent offer and I's non-contingent offer (since the tie-free part of the robustness implies that they do not tender to the non-contingent offers of both) and hence they must prefer these to R's non-contingent offer (*i.e.* $\pi p_R^{\text{sc}} + (1-\pi)\omega_I = p_I^{\text{s}} \geq p_R^{\text{s}}$ and no shares are tendered to R at p_R^{s}). Hence, R's contingent offer is what shareholders tender to, so the non-contingent offer by R is then irrelevant. If R loses with probability 1, then restricting R's strategy space is clearly w.l.o.g. If R wins with probability 1, then replacing any contingent offer with a non-contingent one will not change shareholder or I's behaviour. That the constructed equilibrium is robust is obvious. ‖

Theorem 5 *If $\omega_k + b_k > \omega_j + b_j$, then in all equilibria k wins.*

Proof. The method of the proof is again to rule out asymmetric equilibria in which both contenders win with positive probability. Recall that in such a putative asymmetric equilibrium, the shareholders are just indifferent about tendering to R and exactly half tender to R. We know from the analysis in Section 4.1 that there is no such equilibrium when both contenders make non-contingent offers. We now have to extend this conclusion to the cases in which at least one contender makes a conditional offer and the shareholders are indifferent between such an offer and an alternative.

Consider therefore the case in which R makes a contingent offer p_R^{sc} and I responds with a non-contingent offer p_I^{s}. In an asymmetric equilibrium of the tendering subgame, it may not be that $p_R^{\text{sc}} < \omega_I$ since then this outcome would fail robustness due to Pareto domination by the strict equilibrium in the subgame in which shareholders hold on to their shares. Therefore, $p_R^{\text{sc}} \geq \omega_I$. In an asymmetric equilibrium of the subgame, the shareholders would be indifferent either between tendering to R and tendering to I or between tendering to R and just holding on to the shares. The latter case is ruled out since it implies $\pi p_R^{\text{sc}} + (1-\pi)\omega_I = \pi\omega_R + (1-\pi)\omega_I$, hence $p_R^{\text{sc}} = \omega_R$, which is not consistent with $\pi \in (0, 1)$ and the tie-free condition of robustness.

In the former case, $\pi p_R^{\text{sc}} + (1-\pi)\omega_I = p_I^{\text{s}}$ so that $\pi = \frac{p_I^{\text{s}} - \omega_I}{p_R^{\text{sc}} - \omega_I}$ and $u_I = (1-\pi)b_I + (\pi\omega_R + (1-\pi)\omega_I - p_I^{\text{s}})\theta = b_I + \frac{p_I^{\text{s}} - \omega_I}{p_R^{\text{sc}} - \omega_I}((\omega_R - p_R^{\text{sc}})\theta - b_I)$, where $\theta \leq 1/2$ is the fraction selling to I. Now, if $\left(\frac{\omega_R - p_R^{\text{sc}}}{2} - b_I\right) > 0$, then u_I is increasing in p_I^{s} so I will set $p_I^{\text{s}} = p_R^{\text{sc}}$ resulting in $\pi = 1$. If $\left(\frac{\omega_R - p_R^{\text{sc}}}{2} - b_I\right) < 0$, then u_I is decreasing in p_I^{s} so I will set $p_I^{\text{s}} = \omega_I$ resulting in $\pi = 0$. Thus, in either case $\pi \in \{0, 1\}$.

The rest of the proof is as in the case of non-contingent offers. ‖

Proofs for Section 6.2

Theorem 6 *The efficient contender wins in equilibrium except in the following regions of the parameter space.*

1. If $\omega_I + b_I > \omega_R + b_R$ and $b_R > 2b_I$, then R wins.
2. If $\omega_I + b_I < \omega_R + b_R < \omega_I + 2b_I$ and $b_I > b_R$, then I wins.

Proof. The proof is like that of Theorem 2. It follows from the subsequent characterization of equilibrium outcomes and existence. By Lemma A5 and existence, $\pi \in \{0, 1\}$. Propositions A5 and A6 preclude either $\pi = 0$ or $\pi = 1$ for all possible configurations of the parameters . For example, part 1 follows from Proposition A5 part (ii). ‖

Before proving that in all equilibria $\pi \notin (0, 1)$, we present a result analogous to Lemma A3 showing that for our purposes we can restrict attention to a subset of the strategy space.

Lemma A4. *The equilibrium value of π is unchanged if we restrict attention to the case where I makes only non-contingent offers and R does not make both contingent and non-contingent offers for shares and R also does not make both contingent and non-contingent offers for votes, i.e. $p_R^{\text{v}} \times p_R^{\text{vc}} = 0$ and $p_R^{\text{s}} \times p_R^{\text{sc}} = 0$.*

Proof. The proof follows exactly the same lines as that of Lemma A3. The only change is that if there is an equilibrium in which I offers $p_I^{\text{vc}} > 0$, we must show that there is an alternative equilibrium in which $p_I^{\text{vc}} = 0$. This follows since instead of offering p_I^{vc}, I could offer $p_I^{\text{v}} = (1-\pi)p_I^{\text{vc}}$. When offering p_I^{vc}, the pay-offs to shareholders tendering votes to I conditionally would be $(1-\pi)p_I^{\text{vc}} + (1-\pi)\omega_I + \pi\omega_R$ and the pay-off to I would be $(1-\pi)b_I + \theta(1-\pi)(-p_I^{\text{vc}})$, where $\theta \in [0, 1/2]$ is the fraction of shares tendered to I. With $p_I^{\text{v}} = (1-\pi)p_I^{\text{vc}}$, the same outcome will continue to be an equilibrium of the tendering subgame. This is because given the same π, those tendering to I and those who do not will receive the same pay-off and I's pay-off will be $(1-\pi)b_I + \theta(-p_I^{s\prime}) = (1-\pi)b_I + \theta(1-\pi)(-p_I^{sc})$ just as before. ‖

Lemma A5. *With conditional (but unrestricted) offers for shares and votes, there is no equilibrium in which I and R both have a strictly positive probability of winning, i.e. there is no equilibrium with $\pi \in (0,1)$.*

Proof. For $\pi \in (0,1)$, it must be that shareholders tender shares to one contender and votes to the other.

The tendering of non-contingent shares both to I and to R is precluded by the tie-free part of the robustness. Tendering of non-contingent shares to I and contingent shares to R is precluded by the following argument. If this were the case, we would have $\pi p_R^{sc} + (1-\pi)w_I = p_I^{s}$. It may not be that $p_R^{sc} = w_I = p_I^{s}$ since then the tie-free part of the robustness would rule out tendering to both. So, it has to be either $w_I < p_I^{s} < p_R^{sc}$ or $w_I > p_I^{s} > p_R^{sc}$. But both of these cases are ruled out by the Pareto domination part of the robustness requirement. In the first case, the putative equilibrium outcome in the tendering subgame is Pareto dominated by all tendering to R which is a strict equilibrium in the tendering subgame (note that $p_R^{sc} \geq w_R$ or else there will be no tendering to R in the first place). Consider then the second case and a $(f_R^{\varepsilon}, f_I^{\varepsilon})$ as required by the robustness condition. If $p_I^{v\varepsilon} > p_R^{v\varepsilon}$, then the equilibrium where all shareholders tender votes to I is a strict equilibrium that Pareto dominates the original outcome π. If $p_R^{v\varepsilon} > p_I^{v\varepsilon}$, then it must be that $p_R^{sc} > w_R$ as otherwise it is not an equilibrium for shareholders to sell shares to R as selling votes to R yields more $(\pi p_R^{sc} + (1-\pi)w_I < \pi w_R + (1-\pi)w_I + p_R^{v})$. But then I's profits are $\frac{1}{2}(\pi w_R + (1-\pi)w_I) - \frac{1}{2}p_I^{s} + (1-\pi)b_I$ which equals $\frac{1}{2}\pi(w_R - p_R^{sc}) + (1-\pi)b_I$ (by substituting $\pi p_R^{sc} + (1-\pi)w_I = p_I^{s}$) which is decreasing in π in which case the optimal p_I^{s} is equal to w_I whereupon $\pi = 0$.

The same type of arguments rule out the sale of votes to both I and R.

Finally, there cannot be an equilibrium with $\pi \in (0,1)$ in which some shareholders tender to R and some do not tender at all. The impossibility of some not tendering and some tendering shares for non-contingent prices was demonstrated in Lemma 1. That they cannot be indifferent between selling votes at non-contingent or contingent prices and not tendering is obvious. The possibility of some tendering to a contingent offer by R and some not tendering when $p_R^{cs} = w_R$ is ruled our by the tie-free part of the robustness requirement.

Given that w.l.o.g. contenders neither make both a conditional and an unconditional offer for shares nor make both conditional and unconditional offers for votes, the preceding discussion implies that if $\pi \in (0,1)$, then one of the following must hold:

1. $\pi p_R^{sc} + (1-\pi)w_I = \pi w_R + (1-\pi)w_I + p_I^{v}$.
2. $\pi p_R^{sc} + (1-\pi)w_I = \pi w_R + (1-\pi)w_I + (1-\pi)p_I^{vc}$.
3. $p_R^{s} = \pi w_R + (1-\pi)w_I + p_I^{v}$.
4. $p_R^{s} = \pi w_R + (1-\pi)w_I + (1-\pi)p_I^{vc}$.
5. $\pi p_R^{vc} + \pi w_R + (1-\pi)w_I = p_I^{s}$.
6. $\pi p_R^{vc} + \pi w_R + (1-\pi)w_I = (1-\pi)p_I^{sc} + \pi w_R$.
7. $p_R^{v} + \pi w_R + (1-\pi)w_I = p_I^{s}$.
8. $p_R^{v} + \pi w_R + (1-\pi)w_I = (1-\pi)p_I^{sc} + \pi w_R$.

We consider these cases next. For Cases 1–4, as in Lemma A1, if $w_I > w_R$, then in the tendering subgame the strict equilibrium in which all tender to I (which one can easily verify *is* an equilibrium of the tendering subgame when the relevant equality condition in 1, 2, 3, or 4 is satisfied) Pareto dominates for shareholders any equilibrium of the tendering subgame with $\pi \in (0,1)$. So the robustness requirement implies that $\pi \notin (0,1)$. Hence, in 1–4 we only consider the case $w_I < w_R$.

(i) If $p_R^{sc} > w_R + p_I^{v}$, then all sell to R by the Pareto undomination part of the robustness requirement. If $p_R^{sc} < w_R + p_I^{v}$, then the only equilibrium of the tendering subgame is for all to sell to I. Hence, if $p_R^{sc} \neq w_R + p_I^{v}$, we have $\pi \notin (0,1)$. In the case $p_R^{sc} = w_R + p_I^{v}$, the tie-free part of the robustness implies $\pi \notin (0,1)$.

(ii) Given any equilibrium of this type with some $\pi \in (0,1)$, we can construct an equilibrium of Type 1 with $p_I^{v} = (1-\pi)p_I^{vc}$ since then pay-offs to shareholders and to I and R are the same. Since no equilibrium of Type 1 with $\pi \in (0,1)$ exists, the same conclusion applies to equilibria of Type 2. (There is also a simple direct argument: $p_R^{sc} > w_R$ since otherwise no one sells to R. Since $w_R > w_I$, all selling to R—which is an equilibrium of the tendering subgame—is better than any pay-off with $\pi \in (0,1)$, so by the Pareto undomination part of the robustness requirement $\pi \notin (0,1)$. The case of $\pi \in (0,1)$ arising due to $p_R^{sc} = w_R$ is ruled out by the tie-free part of the robustness requirement.

(iii) This situation is identical to the case studied in Lemma A1 of $\pi \in (0,1)$ without conditional offers and therefore is not feasible for $\pi \in (0,1)$.

(iv) The same argument as in Case 2, but applied to Case 3, implies that there is no equilibrium with $\pi \in (0,1)$ in Case 4.

We turn now to Cases 5–8. As discussed in Lemma A1, $w_R > w_I$ implies that the Pareto undomination part of the robustness requirement selects the equilibrium in the tendering subgame where all sell to R. So we consider $w_I > w_R$.

(v) Assume there is an interior solution for π (otherwise we are done with this step).
If $p_I^{\rm s} < w_R + p_R^{\rm vc}$, then all selling to R is the only equilibrium outcome of the tendering subgame that survives the Pareto undomination part of the robustness requirement.
If $p_I^{\rm s} > w_R + p_R^{\rm vc}$, then since we are assuming there is an interior solution for π, we must also have $w_I > p_I^{\rm s}$ (by the equality in condition 5). Then $\pi = \frac{w_I - p_I^{\rm s}}{w_I - p_R^{\rm vc} - w_R}$ and $u_I = (1-\pi)b_I + (\pi w_R + (1-\pi)w_I - p_I^{\rm s})\theta = (1-\pi)b_I - \pi p_R^{\rm vc}\theta$, where $\theta \leq 1/2$ is the fraction of conditional votes purchased by I. This is decreasing in π and hence increasing in $p_I^{\rm s}$. So the optimal solution for I is at $\pi = 0$.
If $p_I^{\rm s} = w_R + p_R^{\rm vc}$, then by the tie-free part of the robustness requirement $\pi \notin (0,1)$.

(vi) The argument in the proof of Lemma A4 implies that we can assume w.l.o.g. that I does not make conditional price offers. Hence, the proof in part 5 applies to this case. (There is also a simple direct argument: $p_I^{\rm sc} \geq w_I$ since otherwise no one sells to I. Since $w_I > w_R$, all selling to I—which is an equilibrium in the tendering subgame—is better than any pay-off with $\pi \in (0,1)$, so, if $p_I^{\rm sc} > w_I$, by the Pareto undomination part of the robustness requirement refinement $\pi \notin (0,1)$. The case $p_I^{\rm sc} = w_I$ and $\pi \in (0,1)$ is ruled out by the tie-free part of the robustness requirement.)

(vii) This is the same as in the unconditional analysis of Lemma A1.

(viii) The argument in the proof of Lemma A4 again implies that we can assume w.l.o.g. that I does not make conditional price offers. Hence, the proof in part 7 applies to this case. (There is also a simple direct argument: If $p_I^{\rm sc} > w_I + p_R^{\rm v}$, then all sell to I by the Pareto undomination part of the robustness requirement. If $p_I^{\rm sc} < w_I + p_R^{\rm v}$, then the only equilibrium in the tendering subgame is for all to sell to R. Hence, if $p_I^{\rm sc} \neq w_I + p_R^{\rm v}$, we have $\pi \notin (0,1)$. The case of $\pi \in (0,1)$ due to $p_I^{\rm sc} = w_I + p_R^{\rm v}$ is ruled out by the tie-free part of the robustness requirement.) ‖

Proposition A5. *If (i) $w_R + b_R > w_I + 2b_I$ or (ii) $b_R > 2b_I$ or (iii) both $w_I + b_I < w_R + b_R < w_I + 2b_I$ and either $w_I > w_R$ or $b_I < b_R < 2b_I$, then I cannot win in any equilibrium.*

Proof. The proof of parts (i) and (ii) exactly mimics parts A and B in the proof of Proposition A1, except that in addition to considering I responding with $p_I^{\rm v}$ or $p_I^{\rm s}$, we also allow for responses of $p_I^{\rm vc}$ and $p_I^{\rm sc}$. That is, $\pi = 0$ cannot arise in equilibrium since R can open with $p_R^{\rm s} \in (\max\{w_I + 2b_I, w_R\}, w_R + b_R)$ if condition (i) of the proposition holds or with $p_R^{\rm v} > 2b_I$ if condition (ii) of the proposition holds.

That against the former an offer of $p_I^{\rm sc}$ that wins with positive probability is not profitable holds for the same reason that an offer of $p_I^{\rm s}$ that wins with positive probability is not profitable. That an offer of $p_I^{\rm vc}$ that wins with positive probability is not profitable holds since when $p_I^{\rm vc} + w_I \geq p_R^{\rm s}$ if I wins then I has losses because $p_I^{\rm vc} \geq p_R^{\rm s} - w_I > 2b_I$, while if $p_I^{\rm vc} + w_I < p_R^{\rm s}$ all sell to R.

Against $p_R^{\rm v} > 2b_I$ again it is clearly unprofitable for I to win with an offer of $p_I^{\rm vc}$ just as with an offer of $p_I^{\rm v}$. An offer of $p_I^{\rm sc} \geq w_I + p_R^{\rm v}$ and I winning results in I having losses, while $p_I^{\rm sc} < w_I + p_R^{\rm v}$ results in all selling to R.

Similarly, the proof for part (iii) mimics part C in the proof of Proposition A1. To be comprehensive, we repeat it here and note that the same arguments work when I also can respond with $p_I^{\rm vc}$ and $p_I^{\rm sc}$. If $w_I > w_R$, it cannot be that $\pi = 0$. If R offers $p_R^{\rm s} \in (w_I + b_I, w_R + b_R)$, then I has no profitable counter-offer and R has profits. To see that I has no profitable counter-offer, first note that $p_I^{\rm s} > p_R^{\rm s}$ can only lead to losses, and the same holds for $p_I^{\rm sc}$. (If $p_I^{\rm s} = p_R^{\rm s}$ and I wins profitably, then some, but not all, shareholders sell to I, but this is ruled out by the tie-free part of the robustness requirement.) If $p_I^{\rm v} < p_R^{\rm s} - w_I$, then $\pi = 1$. If $p_I^{\rm v} > p_R^{\rm s} - w_I$, then all shareholders tender to I and $u_I = b_I - p_I^{\rm v} < b_I + w_I - p_R^{\rm s} < 0$. (If $p_I^{\rm v} = p_R^{\rm s} - w_I$ and I wins profitably, then some, but not all, shareholders sell to I, but this is ruled out by the tie-free part of the robustness requirement.) The same holds for $p_I^{\rm vc}$.

If $w_R > w_I$, then it cannot be that $\pi = 0$. If R offers $p_R^{\rm v} \in (b_I, b_R)$, then I has no profitable counter-offer and R has profits. To see that I has no profitable counter-offer, first note that $p_I^{\rm v} \geq p_R^{\rm v}$ and I winning can only lead to losses for I, and the same for $p_I^{\rm vc}$. If $p_I^{\rm s} < w_R + p_R^{\rm v}$, then (due to the Pareto undomination part of the robustness requirement) I loses. If $p_I^{\rm s} > w_R + p_R^{\rm v}$, then all shareholders sell to I and I has losses, and the same holds for $p_I^{\rm sc}$. (If $p_I^{\rm s} = w_R + p_R^{\rm v}$ and I wins profitably, then some, but not all, shareholders sell to I, but this is ruled out by the tie-free part of the robustness requirement.) ‖

Proposition A6. *If $w_R + b_R < w_I + b_I$ and $b_R < 2b_I$ or $b_R < b_I$ and $w_R + b_R < w_I + 2b_I$, then R cannot win.*

Proof. The proof mimics that of Proposition A2. The only difference is that R may open with $p_R^{vc} \geq b_I$. In this case, setting $p_I^s = w_I + p_R^{vc}$ (analogous to the behaviour after $p_R^v \geq b_I$) is not profitable for I as due to the contingent nature of R's offer, all will tender to I. However, we have that $p_R^{vc} < b_R$ (since otherwise if R wins with probability 1, then R has losses), and then if I sets p_I^s just above $w_R + p_R^{vc}$, everyone sells to I and this is profitable to I. ‖

Remark A1. The parameter regions considered in Propositions A5 and A6 include all possible configurations, but they are not a partition of the parameter space, *e.g.* (i) and (ii) of Proposition A5 overlap.

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