On Evidence in Games and Mechanism Design

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Based on joint work with:

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and

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Evidence

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  - the public sphere (courts / government decisions); within organizations.
- **Hard evidence** about types does play a significant role in these and other contexts.
- Today: briefly review some models with evidence
  - Not a detailed survey; partial overview
  - Emphasis on 3 papers w/ Ben-Porath and Lipman

A partial taxonomy

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One can consider mechanism-design problems or games without commitment.

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This setting is common in the literature: the seller/agent wants to be thought to have high value (type-independent preferences), and the buyer/principal wants to learn the actual value.

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Let v^* be the belief about the seller in the absence of evidence.

Clearly any type with value $v < v^*$ will not present evidence.

Let $v^* = E\left[v : v \text{ has no evidence or } v < v^*\right]$; v^* is unique.

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Equilibrium:

Types with value above v^* present evidence of their type.
Types with value below v^* pool with no-evidence types.

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Richer evidence models – Dye evidence

This is a workhorse model in accounting in particular, and economics more generally.

Some recent contributions in economics include, e.g., Shin (2003) and Archarya, DeMarzo, and Kremer (2011) among *many* others, and work that I'll present later.

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For example, Shin shows how in the Dye evidence structure strategic disclosure is consistent with data on stock price variability and rates of return being higher after bad news / low prices.

Archarya et. al. show how such an environment can lead to clustering of bad news announcements (but not good news), and how this is also consistent with data on price variability.

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Dye evidence: applications – Shin (2003)

- There are n projects, each has value H with prob r and L otherwise.
- Two periods:
  - 1: Manager observes outcome of each project iid with prob q, and may reveal some outcomes.
  - 2: Value of firm determined.

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- Two periods:
  - 1: Manager observes outcome of each project iid with prob q, and may reveal some outcomes.
  - 2: Value of firm determined.
- For simplicity assume $n = 1$. Contrast equilibrium where:
  - manager only reveals a success if it is observed, vs.
  - all available information is (exogenously) revealed.

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  - lower prices are followed by greater future-price uncertainty.

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  - after success revealed: both periods' price is H; no residual uncertainty
  - after nothing revealed: period-1 price lower; uncertainty over future price.
  - lower prices are followed by greater future-price uncertainty.
- With exogenous disclosure the lowest price is when failure revealed, and then no subsequent price uncertainty:
  - non-monotonicity in residual uncertainty as function of period-1 prices.
- Former broadly consistent with data, suggesting that strategic disclosure plays a significant role.

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Highlights the importance of understanding the strategic disclosure environment for disclosure policy.

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In addition to the substantive interest, this evidence structure illustrates an important feature: seller can present evidence *e or* e' but not both *e and* e'; e.g., due to limited attention.

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This assumption, called *normality*, plays an important role in the analysis of disclosure.

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Other evidence and economic models

- In Verrecchia (1983) agents have costs of presenting evidence.
    - Low-value types will not find it worthwhile to present evidence.
    - So again have only partial unraveling.
    - Also widely applied, but I'll focus more on Dye's model.

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  - Also widely applied, but I'll focus more on Dye's model.
- Farrell (1986), Matthews and Postlewaite (1985), Okuno-Fujiwara, Postlewaite and Suzumura (1990) are some other classical models with different evidence environments.

How is evidence modeled?

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Normality: Any t can present all his evidence:

$$\forall t : M(t) \equiv \cap_{e \in E(t)} e \in E(t).$$

Disclosure and choice (BDL)

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Here voluntary disclosure can lead to significant efficiency loss.

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Focus on risk-taking as an illustration of the possible distortions.

An agent can gamble on having something positive to show, but hide his information if things go badly.

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Benchmark

As before, we contrast the strategic disclosure environment with the benchmark of exogenous (or mandatory) disclosure.

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As we'll see the same will occur if the evidence is obtained with probability 1 (unraveling) or there is no evidence (no ability to distort).

Thus, this efficient outcome is a natural benchmark from several perspectives.

Disclosure and choice (BDL)

Examples

- Agent is manager, observer is the market.
    - In the long run the outcome of the manager's actions is realized and determines the firm's value.
    - In the short run can release private information about success.
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- Agent is incumbent politician, observer is representative voter.

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- If so expected outcome – even if no evidence presented – is 4.
- But then agent's payoff to deviating to F_2 is

$$q\left[\frac{1}{2}(4) + \frac{1}{2}(6)\right] + (1-q)(4) > 4.$$

For $q > 1/2$ F_2 is not an equilibrium

Disclosure and choice (BDL)

Results

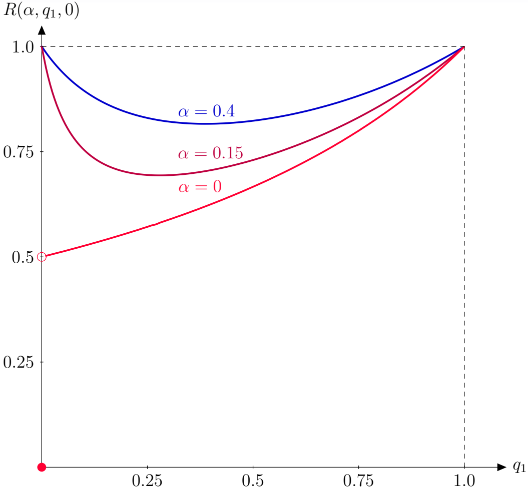
Two Theorems for Base Model:

1. Agent engages in excessive risk taking.
2. Payoffs can be as low as 50% of the first best, but no lower.

▸ Proof of lower bound

▸ Proof lower bound attained

▸ Solving for the equilibrium



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$R(\cdot)$ is % of first-best attained in worst-case equilibrium as $\mathcal{F}$ is varied

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A challenger

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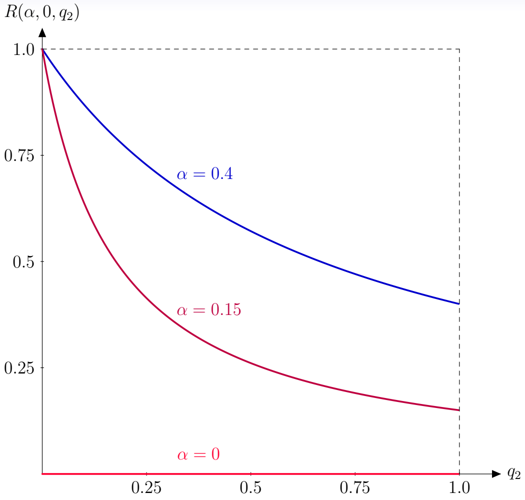
Not completely "symmetric". Example:

Let F give 0 or 3 with equal probability, and let G give -1 or 100.

If F is expected, then without evidence belief is between 0 and 3.

So a deviation is harmful (as -1 will be presented but 100 will not).



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Incumbent and challenger

Efficiency iff challenger and incumbent have "equal" access to evidence, i.e., their probabilities of having perfect evidence are equal.

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Separation results

Multiple senders with conflicting preferences can lead to separation under much weaker assumptions about the economic environment, the rationality of the principal and the evidence structure.

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For example, Milgrom and Roberts (1986) show how conflicting interests among senders can retain the fully separating equilibrium even when the receiver has limited rationality and with more general preferences of senders.

Lipman and Seppi (1995) showed how the same separation results under conflicting interests with much weaker evidence structures.

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Separation results: Lipman and Seppi example

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Nevertheless can have full separation: the competitor states the quality and the seller refutes it if he understates, and after refuting the observer believes it is the best quality.

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See Green and Laffont (1986), Bull and Watson (2007), Deneckere and Severinov (2008), and Forges and Koessler (2005).

Mechanism design: single agent

On commitment and randomization

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- Sher (2011): allows for multiple actions
  - $3'$: $\forall t \; v(a, t) = f_t(u(a))$, where f_t is concave.

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- Sher (2011): allows for multiple actions
  - $3'$: $\forall t \;\, v(a,t) = f_t(u(a))$, where f_t is concave.
- Hart, Kremer and Perry (2016): insightful proof
  - $3''$: concavification of avg of f_t's equals avg of concavifications.

Mechanism design: single agent

On commitment and randomization: example

- Seller may be of type with value $v \in T = \{1, \ldots, 999\}$.
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- Evidence:
  - $E(999) = \{T\}$, $E(i) = \{\{i\}, T\}$ for $i < 999$.
- Principal wants to guess value.
  - Loss function strictly increasing in the distance between guess a and true type.

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- If loss is 0 when correct and 1 when wrong then commitment has value:
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    - In equilibrium without commitment, after observing no evidence principal chooses 999.
      So no one presents evidence and the principal's payoff is .002.

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- If loss is 0 when correct and 1 when wrong then commitment has value:
  - In equilibrium without commitment, after observing no evidence principal chooses 999.
    So no one presents evidence and the principal's payoff is .002.
  - If principal commits to guessing 1 when no evidence is presented get perfect revelation of all types except 999.

Mechanism design: single agent

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    So payoff is .998.

Mechanism design: multiple agents

On commitment, randomization and robustness (BDL)

Simple allocation problem.

Principal allocates indivisible good to one of I agents: $A = I$.

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Each agent wants the good; $u_i(i) = 1, u_i(j) = 0, j \neq i$.

Value to principal of giving good to i depends on t_i, and is $v_i(t_i)$

$$\sum_i u_i(a) v_i(t_i)$$

On commitment, randomization and robustness (BDL)

Examples and extensions

- Examples:
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On commitment, randomization and robustness (BDL)

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- Binary decision $A = \{0, 1\}$ with general type-dependent preferences:
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  - Principal's preferences: $\sum u_i(a, t_i)v_i(t_i)$

▸ Public good example

On commitment, randomization and robustness (BDL)

Results

For a class of mechanism-design problems (including those above):

- Randomization has no value for the principal.

On commitment, randomization and robustness (BDL)

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- Randomization has no value for the principal.
- Requiring *robust IC* (instead of Bayesian IC) has no cost for the principal.
- Commitment has no value for the principal.
  - The equilibrium that is equivalent to the optimal mechanism comes from a simple alternative game.
  - It may involve mixed strategies by the agents.
  - Can use this to characterize optimal mechanisms by finding the best equilibrium of the game.

▸ Simple type dependence

On commitment, randomization and robustness (BDL)

Robust incentive compatibility

Bayesian IC: Honesty is optimal in expectation given honesty by others.

On commitment, randomization and robustness (BDL)

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On commitment, randomization and robustness (BDL)

Robust incentive compatibility

Bayesian IC: Honesty is optimal in expectation given honesty by others.

Robust IC: Honesty is optimal no matter what other agents' types are and no matter what they do.

Stronger than dominant-strategy IC and than ex-post IC.

On commitment, randomization and robustness (BDL)

Commitment and robust PBE

- Game:
  - Agents simultaneously send reports (cheap talk) and evidence to the principal.
  - The principal chooses $p \in \Delta(A)$. [No commitment.]

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    (Can be extended to general case we analyze.)

Proof Sketch

On commitment, randomization and robustness (BDL)

Optimal mechanisms with Dye evidence: equilibrium

Hence can describe optimal mechanisms using equilibrium of artificial game.

On commitment, randomization and robustness (BDL)

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On commitment, randomization and robustness (BDL)

Optimal mechanisms with Dye evidence: equilibrium

Hence can describe optimal mechanisms using equilibrium of artificial game.

With Dye evidence, straightforward to find i's equilibrium behavior.

Focus on simple allocation problem where each i wants the principal to believe v_i is large.

On commitment, randomization and robustness (BDL)

Optimal mechanisms with Dye evidence: equilibrium

- The equilibrium is as before. There exists unique v_i^* s.t.:
  - $v_i^* = \mathrm{E}[v_i(t_i) : t_i$ has no evidence or $v_i(t_i) < v_i^*]$.
  - If i doesn't present evidence, principal's belief is v_i^*.
  - Every type t_i with evidence and with $v_i(t_i) \geq v_i^*$ shows his evidence and no other type shows evidence.

On commitment, randomization and robustness (BDL)

Optimal mechanisms with Dye evidence: application of characterization

This makes it easy to characterize an optimal mechanism.

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For any t_i let $\hat{v}_i(t_i)$ be the principal's equilibrium beliefs about v_i:

$$\hat{v}_i(t_i) = \begin{cases} v_i^*, & \text{if } t_i \text{ has no evidence or } v_i(t_i) < v_i^*; \\ v_i(t_i), & \text{otherwise.} \end{cases}$$

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In the optimal mechanism, given profile t, the principal allocates the good to the type with the highest $\hat{v}_i\left(t_i\right)$.

On commitment, randomization and robustness (BDL)

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For simplicity, assume two agents and $v_1^* > v_2^*$.

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$\hat{v}_1(t_1) \geq v_1^*$ for all t_1.

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This is a *favored-agent mechanism* where 1 is favored and v_1^* is threshold.

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Can extend this method to the other economic environments mentioned.

▸ Extensions

Costly verification

Single agent

- Until now assumed evidence is costless to receive and the agent determines whether it is provided.

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- Townsend (1979): Costly State Verification.
  - Principal-agent model, where the principal can check the type of the agent at a cost.
  - Focus is on optimal contracting.

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  - Principal-agent model, where the principal can check the type of the agent at a cost.
  - Focus is on optimal contracting.
  - See also Gale and Hellwig (1985), Border and Sobel (1987), Mookherjee and Png (1989).
- Glazer and Rubinstein (2004):
  - Principal-agent model where the type has two dimensions.
  - Cost of verifying one dimension is zero, but both is infinite.
  - Randomization *is* needed in the optimal mechanism.

Costly verification (BDL)

Multiple agents

We consider the simple allocation problem discussed earlier:

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Principal has one good to allocate.

Agents have privately-known types with values v_i, where v_i is the value to the principal from giving the good to i.

All agents want the good.

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Agents have privately-known types with values v_i, where v_i is the value to the principal from giving the good to i.

All agents want the good.

Can allow for a reservation value to the principal by adding dummy player with that value.

Costly verification (BDL)

Multiple agents

No monetary transfers.

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Principal can pay cost c_i to obtain perfect evidence about i's type.

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The question is how to optimally use the costly verification option.

Obviously when costs are zero can check everything, and when they are very high will just allocate by the prior. In between?

Costly verification (BDL)

Favored-agent mechanism (again!)

We show that the optimal Bayesian IC mechanism is again a favored-agent mechanism:

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It specifies a favored agent i^* and a threshold value v^* such that

- If every non-favored agent reports a type with value below v^*, then i^* gets the object and no verification is carried out.
- Otherwise, agent with highest reported value is checked and gets the object if (as will happen in equilibrium) report is found to be correct.

Costly verification (BDL)

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The favored-agent mechanism

- As before the mechanism satisfies robust IC and is deterministic.
- However here commitment is being used, as the principal has no reason to check ex post.
- This connection between Dye evidence and costly verification is more general.
  - The optimal mechanisms for both have the same structure in other cases:
  - The binary-choice model (using the characterization from before, and Erlanson and Kleiner (2016)).
  - The k–good (or bad) allocation problem.

Costly verification (BDL)

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    - This is favored agent with $\hat{\imath}$ and threshold c.
    - Can improve by choosing $\hat{\imath}$ and threshold optimally.

Costly verification (BDL)

Finding the favored agent and threshold

Assuming i is favored we can calculate the threshold v_i^* such that the principal is indifferent between treating $\max_{j \neq i} v_j$ as just above or below the threshold.

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Another interesting connection: the equation above is also used to determine the optimal strategy in a Weizmann-like search problem where "boxes" can be taken without being opened (Doval, 2014).

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- How verification costs affect the optimal mechanism.

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- Exploring how evidence and costly verification are useful in different environments and when interacting with other tools such as transfers.
- Exploring the similarity between the structure of the optimal mechanism under Dye evidence and costly verification.

THANK YOU

If beliefs are that the manager chooses F_2 and he does so then he gets 3. If he deviates to F_1 he gets

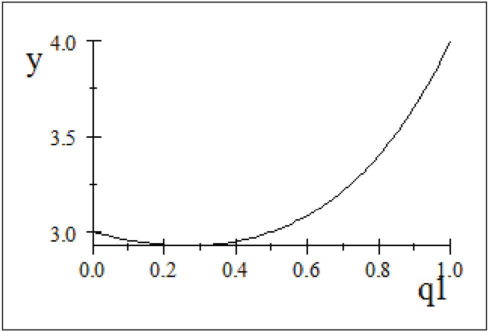
$$\alpha 3 + (1-\alpha)(q_1 4 + (1-q_1)\hat{x}(F_2))$$

where

$$\hat{x}(F_2) = ((1-q_1)3 + 0)/(1 - q_1 + q_1/2)$$

So deviating gives

$$4q_1 + 3(1-q_1)^2/(1-q_1/2)$$



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Since $q_1 > 0$, $F \in \text{supp } \sigma$, and σ is optimal:

$$\mathrm{E}_F \max\{x, \hat{x}\} \geq \mathrm{E}_G \max\{x, \hat{x}\}$$

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Since $F(x) \leq 1$ and $G(x) \geq 0$, this requires

$$\mathrm{E}_F(x) + \hat{x} \geq \mathrm{E}_G(x).$$

Proof of bounds on inefficiency (cont'd)

Since σ is an equilibrium,

$$\sum_{F' \in \mathcal{F}} \sigma(F') \mathrm{E}_{F'}(x) = (1 - q_1)\hat{x} + q_1 \sum_{F' \in \mathcal{F}} \sigma(F') \mathrm{E}_{F'} \max\{x, \hat{x}\}.$$

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Since $\mathrm{E}_{F'} \max\{x, \hat{x}\} \geq \mathrm{E}_{F'}(x)$,

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Since $\mathrm{E}_{F'} \max\{x, \hat{x}\} \geq \mathrm{E}_{F'}(x)$,

$$\sum_{F' \in \mathcal{F}} \sigma(F')\mathrm{E}_{F'}(x) \geq \hat{x}.$$

Since $\mathrm{E}_F(x) \leq \mathrm{E}_{F'}(x)$ for all $F' \in \text{supp } \sigma$

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Proof of bounds on inefficiency (cont'd)

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So: $2 \sum_{F' \in \mathcal{F}} \sigma(F')\mathrm{E}_{F'}(x) \geq \mathrm{E}_F(x) + \hat{x} \geq \mathrm{E}_G(x).$

Back

Bounds on inefficiency

Proof that the 1/2 bound is achievable

$\mathcal{F} = \{F, G\}$
F : probability $1 - p$ on 0 and p on $1/p$, so $\mathrm{E}_F(x) = 1$
G : probability 1 on $x = x^*$.
In equilibrium F will be chosen when $x^* \approx 2$.

Bounds on inefficiency

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G : probability 1 on $x = x^*$.
In equilibrium F will be chosen when $x^* \approx 2$.

If the observer expects the agent to choose F with probability 1, then $\hat{x}$ solves

$$(1 - q_1)\hat{x} + q_1 \left[(1 - p)\hat{x} + 1\right] = 1$$

so

$$\hat{x} = \frac{1 - q_1}{1 - q_1 p}.$$

Bounds on inefficiency

Proof that the 1/2 bound is achievable (cont'd)

F is an equilibrium iff $\mathrm{E}_G \max\{x, \hat{x}\} \leq \mathrm{E}_F \max\{x, \hat{x}\}$ or

$$\max\{x^*, \hat{x}\} \leq (1-p)\hat{x} + 1 = \frac{2 - q_1 - p}{1 - q_1 p}.$$

Bounds on inefficiency

Proof that the 1/2 bound is achievable (cont'd)

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Clearly $\hat{x} < 1$ while we will set, $x^* > 1$. So equilibrium iff

$$x^* \leq \frac{2 - q_1 - p}{1 - q_1 p}.$$

Let x^* equal the RHS, let q_1, $p \approx 0$, so $x^* \approx 2$.

Bounds on inefficiency

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Let x^* equal the RHS, let q_1, $p \approx 0$, so $x^* \approx 2$.

So the agent's payoff is arbitrarily close to half the first-best payoff.

Back

The equilibrium

Solving for equilibrium:

Consider Period 2 (Short Run). If manager gets no information, he can't reveal anything. Let $\hat{x}$ be the stock price in response to that.

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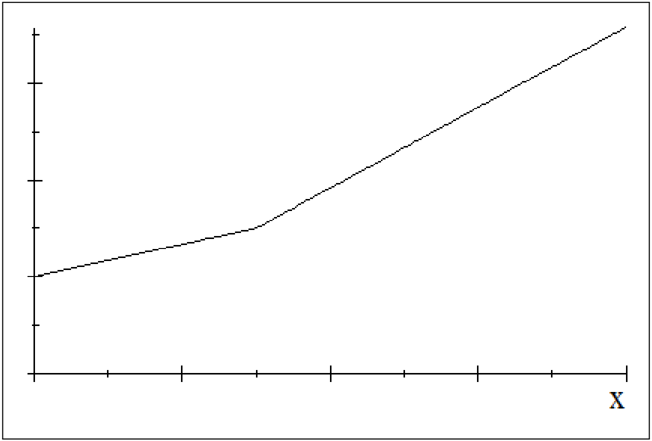
Hence he reveals iff $x \geq \hat{x}$.

The equilibrium

Payoff is

$$\alpha x + (1-\alpha)\left[q_1 \max\left\{x, \hat{x}\right\} + (1-q_1)\hat{x}\right]$$



The equilibrium

To finish constructing equilibrium, we need to characterize $\hat{x}$ and manager's choice of F.

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Given F, we must have

$$\begin{aligned}\hat{x} &= \mathrm{E}_F[x \mid (\text{observed and } x \leq \hat{x}) \text{ OR } (\text{not observed})] \\ &= \frac{(1-q)\mathrm{E}_F(x) + qF(\hat{x})\mathrm{E}_F(x \mid x \leq \hat{x})}{1 - q + qF(\hat{x})}\end{aligned}$$

Determines $\hat{x}$ uniquely given F — call this $\hat{x}(F)$.

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Determines $\hat{x}$ uniquely given F — call this $\hat{x}(F)$.

Given $\hat{x}$, manager chooses F to maximize

$$\alpha \mathrm{E}_F(x) + (1-\alpha)\left[q_1 \mathrm{E}_F \max\{x, \hat{x}\} + (1-q_1)\hat{x}\right].$$

The equilibrium

Easy to show that

$$\alpha \mathrm{E}_F(x) + (1-\alpha)\left[q_1 \mathrm{E}_F \max\{x, \hat{x}(F)\} + (1-q_1)\hat{x}(F)\right] = \mathrm{E}_F(x).$$

The equilibrium

Easy to show that

$$\alpha \mathrm{E}_F(x) + (1-\alpha)\left[q_1 \mathrm{E}_F \max\{x, \hat{x}(F)\} + (1-q_1)\hat{x}(F)\right] = \mathrm{E}_F(x).$$

If players believe F will be played, so that $\hat{x}(F)$ are the beliefs if nothing is observed, then the payoffs from a deviation to G are

$$\alpha \mathrm{E}_G(x) + (1-\alpha)\left[q_1 \mathrm{E}_G \max\{x, \hat{x}(F)\} + (1-q_1)\hat{x}(F)\right]$$

The equilibrium

If $\alpha = 0$ or $\mathrm{E}_F(x) = \mathrm{E}_G(x)$ then deviating from F to G is profitable if

$$\mathrm{E}_F \max\{x, \hat{x}(F)\} < \mathrm{E}_G \max\{x, \hat{x}(F)\}.$$

The equilibrium

If $\alpha = 0$ or $\mathrm{E}_F(x) = \mathrm{E}_G(x)$ then deviating from F to G is profitable if

$$\mathrm{E}_F \max\{x, \hat{x}(F)\} < \mathrm{E}_G \max\{x, \hat{x}(F)\}.$$

Since $\max\{x, \hat{x}\}$ is convex in x the manager does not maximize expected returns; indeed he is risk loving.

Back

Introduction

"Public-good" problem.

Principal decides whether or not to provide some good; $A = \{0, 1\}$.

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Each agent may or may not want the good and has utility $a\tilde{v}_i(t_i)$.

Value to principal of provision of good is sum of utilities.

$$\begin{aligned} v(a,t) &= a\Sigma_i \tilde{v}_i(t_i) \\ &= \Sigma_i \left[u_i(a,t_i) \times |\tilde{v}_i(t_i)|\right] \\ &\text{where } u_i(a,t_i) \in \{-1, 0, 1\} \end{aligned}$$

Back

Results

Definition. u_i exhibits *simple type dependence* if $u_i(a, t_i) = \alpha_i(a)\beta_i(t_i)$.

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Less general than it may appear! An equivalent model: Partition T_i into T_i^+ and T_i^- such that

$$u_i(a, t_i) = \begin{cases} u_i(a), & \text{if } t_i \in T_i^+; \\ -u_i(a), & \text{if } t_i \in T_i^-, \end{cases}$$

and

$$v(a, t) = \sum_i u_i(a) v_i(t_i).$$

Results

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But more general than that may sound:

- Obviously holds with type independent utility.
- Simple type dependence holds in the public goods problem.
- Simple type dependence is without loss of generality if the principal has only two actions.
- Simple type dependence is without loss of generality in any allocation problem without externalities — i.e., one where each agent only cares whether he gets the good or not.

Back

On commitment, randomization and robustness (BDL)

Proof sketch for simple allocation problem:

Formal model:

- $A =$ finite set of allocations/actions available to the principal.
- $\mathcal{I} = \{1, \ldots, I\} =$ set of agents.
- $T_i =$ finite set of types of agent i; independently distributed.
- $u_i : A \to \mathbf{R}$. Agent i's utility function over A.
- Principal's utility $v(a, t) = \sum_i u_i(a) v_i(t_i)$
  - $v_i(t_i)$ is weight the principal puts on i's utility when i is type t_i.
  - $v_i(t_i)$ measures overlap of interest between agent i and the principal.

On commitment, randomization and robustness (BDL)

Mechanisms and Incentive Compatibility

A mechanism gives a probability distribution over A as a function of type reports and evidence presentation by each agent.

$p : T \times E \to \Delta(A)$.

The principal chooses p to maximize

$$\mathrm{E}_t \left[\sum_{a \in A} p(a \mid t, M(t)) v(a, t) \right]$$

subject to *incentive compatibility*.

On commitment, randomization and robustness (BDL)

Mechanisms and Incentive Compatibility

Given a mechanism p, let

$$\hat{u}_i(s, e \mid t_i, p) = \sum_{a \in A} p(a \mid s, e) u_i(a, t_i).$$

Incentive compatibility: Honest reports and providing maximal evidence is optimal:

$$\begin{aligned} &\mathrm{E}_{t_{-i}} \hat{u}_i(t_i, M_i(t_i), t_{-i}, M_{-i}(t_{-i}) \mid t_i, p) \\ &\qquad \geq \mathrm{E}_{t_{-i}} \hat{u}_i(s_i, e_i, t_{-i}, M_{-i}(t_{-i}) \mid t_i, p) \end{aligned}$$

whenever $e_i \in E_i(t_i)$.

On commitment, randomization and robustness (BDL)

Proof sketch

Fix an optimal mechanism.

Agent i only cares about probability he gets the good:

$$\hat{p}_i(s_i, e_i) = \mathrm{E}_{t_{-i}} p(a = i \mid s_i, e_i, t_{-i}, M_{-i}(t_{-i})).$$

Let Π_i be the partition of $T_i \times \mathcal{E}_i$ according to equality under $\hat{p}_i$.

Key step, explained below: We can take p to be measurable wrt Π.

That is, $\hat{p}_i(s_i, e_i) = \hat{p}_i(s_i', e_i')$ implies

$$p(s_i, e_i, s_{-i}, e_{-i}) = p(s_i', e_i', s_{-i}, e_{-i}), \quad \forall (s_{-i}, e_{-i}).$$

On commitment, randomization and robustness (BDL)

Proof sketch

Implication: *Any* mechanism measurable wrt Π sufficiently close to p must be incentive compatible.

Reason: Indifference between reports is preserved by measurability. If close enough, strict preference between reports is preserved.

Hence outcome of mechanism is optimal ex post conditional on each event of Π.

Reason: If not, then shift the mechanism slightly towards the conditionally optimal ex post action. This is an improvement, and as just argued it is IC.

On commitment, randomization and robustness (BDL)

Proof sketch

- Ex post optimality of the principal's action on each event of Π "implies" randomization is not needed.
  - Implies there is an optimal pure best reply; still need to show IC still satisfied.

On commitment, randomization and robustness (BDL)

Proof sketch

- Ex post optimality of the principal's action on each event of Π "implies" randomization is not needed.
  - Implies there is an optimal pure best reply; still need to show IC still satisfied.
- It also implies robust IC:
  - Ex post optimality on each event of Π means allocating the good to the agent who is believed to have the highest type (i for whom $\mathrm{E}(v_i(t_i)|t_i \in \Pi_i)$ is maximal). So the agent wants the belief about him to be as high as possible, regardless of the principal's beliefs about others' types.

On commitment, randomization and robustness (BDL)

Proof sketch

What about no commitment?

Ex post optimality on an event in Π doesn't imply ex post optimality for each possible profile of evidence received by the principal in that event.

It also doesn't imply ex post optimality for messages not sent in the mechanism.

But, if we can overcome these issues and construct equilibrium strategies for agents where the information the principal receives is the same as in events of Π, we've shown commitment not needed.

On commitment, randomization and robustness (BDL)

Proof sketch

We construct these strategies by means of the artificial game: We construct a particular equilibrium and show how to use it to construct an equilibrium of the "real game."

Intuition:

- In the artificial game the agent wants to convince the principal he has as high as possible $\mathrm{E}[v_i(t_i) \mid \mathrm{reports}]$.
- In the "real" game the principal gives the good to the agent with highest $\mathrm{E}[v_i(t_i) \mid \mathrm{reports}]$.

Hence in both each agent i wants to persuade principal $v_i(t_i)$ is big, independently of what others are saying.

On commitment, randomization and robustness (BDL)

Proof sketch

Finally, roughly speaking, this is the same as in the optimal mechanism.

First, because in the game the agent wants to convince the principal that he is as high a value type as possible and IC in the mechanism says that as well.

Second the principal could not have more useful information to which she is responding optimally in the game, as this could be done in the mechanism as well and improve the payoff there, a contradiction.

On commitment, randomization and robustness (BDL)

Proof sketch

Why doesn't this work in the usual model? Because "key step" above (measurability wrt Π) doesn't work.

Key step: If t_i indifferent between honest reporting and a lie, we can take the mechanism to give the same outcome for both.

Proof with one agent and two types, t_1 and t_1': Let λ be probability of t_1, suppose a used for t_1 and a' for t_1'.

Suppose principal changes to a with probability λ and a' with probability $1 - \lambda$ for both types.

Now measurable and incentive compatible.

How does principal's expected payoff change?

On commitment, randomization and robustness (BDL)

Proof sketch

Original:

$$\lambda v(a, t_1) + (1-\lambda)v(a', t_1') = \lambda u_1(a)v_1(t_1) + (1-\lambda)u_1(a')v_1(t_1')$$
$$= u_1(a)\left[\lambda v_1(t_1) + (1-\lambda)v_1(t_1')\right]$$

because t_1 is indifferent between lying and not, so $u_1(a) = u_1(a')$.

So principal is indifferent between having a with t_1 and a' with t_1' or the reverse or randomizing.

So principal's utility is unchanged in new mechanism.

Back

Optimal mechanisms with Dye evidence: extensions

Less Simple Allocation Problems:

Example 1: Principal has two units.

Again, agent *i*'s utility is 1 if he receives a unit, 0 otherwise.

Result: Hierarchy of favored agents.

If $I = 3$ and $v_1^* > v_2^* > v_3^*$, then

- 1 gets unit unless *both* 2 and 3 prove types above v_1^*
- 2 gets unit unless 3 proves type above v_2^*

Optimal mechanisms with Dye evidence: extensions

Example 2: Principal has to allocate a "bad": picking department chair.

Again the principal picks agent to give the "good" to.

Agent i's utility is -1 if he gets the good, 0 otherwise.

Let principal's utility to choosing i be $v_i(t_i)$ (a slight notation change). So i wants principal to think $v_i(t_i)$ is small.

Similar structure: With $I = 2$ and $v_1^* > v_2^*$, 1 is chair unless he can prove his competence is below v_2^*.

Optimal mechanisms with Dye evidence: extensions

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1. Positive and negative types separate.

Optimal mechanisms with Dye evidence: extensions

B. Type Dependent Utility: $T_i^+ \neq \emptyset$, $T_i^- \neq \emptyset$.

Again, can easily characterize information revealed in equilibrium in artificial game. Two possibilities:

1. Positive and negative types separate.

$$v_i^+ = \mathrm{E}[v_i(t_i) \mid t_i \in T_i^+ \text{ and either } t_i \in T_i^n \text{ or } v_i(t_i) < v_i^+]$$

$$v_i^- = \mathrm{E}[v_i(t_i) \mid t_i \in T_i^- \text{ and either } t_i \in T_i^n \text{ or } v_i(t_i) > v_i^-]$$

Again, v_i^+ and v_i^- uniquely defined. If $v_i^- \leq v_i^+$, this is equilibrium.

Optimal mechanisms with Dye evidence: extensions

2. Positive and negative types pool.

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\end{aligned}
$$

Again, v_i^* is uniquely defined. If separation by sign is not possible, then this is equilibrium.

Can define $\hat{v}_i$ and outcome of optimal mechanism analogously to type independent case.

Optimal mechanisms with Dye evidence: extensions

Public Goods Problem:

Focus on case where positive and negative types separate.

Here

$$\hat{v}_i(t_i) = \begin{cases} v_i^+, & \text{if } t_i \in T_i^+ \cap T_i^n \text{ or } t_i \in T_i^+ \setminus T_i^n \text{ and } v_i(t_i) < v_i^+; \\ v_i(t_i), & \text{if } t_i \in T_i^+ \setminus T_i^n \text{ and } v_i(t_i) \geq v_i^+; \\ v_i^-, & \text{if } t_i \in T_i^- \cap T_i^n \text{ or } t_i \in T_i^- \setminus T_i^n \text{ and } v_i(t_i) > v_i^+; \\ v_i(t_i), & \text{if } t_i \in T_i^- \setminus T_i^n \text{ and } v_i(t_i) \leq v_i^+; \end{cases}$$

Optimal mechanism provides public good for t such that $\sum_i \hat{v}_i(t_i) > 0$.

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Optimal mechanism provides public good for t such that $\sum_i \hat{v}_i(t_i) > 0$.

Form exactly parallels Erlanson and Kleiner's optimal mechanism for public goods with costly verification. (Similar result for pooling case.)

Back

Costly verification

The favored agent and threshold

Intuition: Suppose i is favored. Compare thresholds τ and t_i^*, where $\tau > t_i^*$.

Let x be highest report of agent other than i.

	$x < v_i^* < \tau$	$v_i^* < x < \tau$	$v_i^* < \tau < x$
v_i^*	$\mathrm{E}(t_i)$	$\mathrm{E}\max\{t_i, x\} - c$	$\mathrm{E}\max\{t_i, x\} - c$
τ	$\mathrm{E}(t_i)$	$\mathrm{E}(t_i)$	$\mathrm{E}\max\{t_i, x\} - c$

$x > v_i^*$ implies

$$\mathrm{E}\max\{v_i, x\} - c > \mathrm{E}\max\{v_i, v_i^*\} - c = \mathrm{E}(v_i).$$

So v_i^* is a better threshold than τ.

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