

Simultaneous Offers and the Inefficiency of Bargaining: A Two-Period Example

EDDIE DEKEL*

*Department of Economics, University of California,
Berkeley, California 94720*

Received October 25, 1988; revised August 10, 1989

It is shown that the Pareto optimal outcomes in a two period simultaneous move bargaining model violate forwards induction rationality when the players are sufficiently patient. This bargaining model describes a situation where a principal is represented by an agent whose flexibility is restricted. Hence, a bargaining process with such agents can create costly delays. The result also provides another example of the power of forwards induction and stability. *Journal of Econometric Literature*
Classification Number: 026. © 1990 Academic Press, Inc.

1. INTRODUCTION

In the recent literature on noncooperative bargaining, starting from Rubinstein's [25] seminal paper, players make *alternating* offers in sequence, until agreement is reached about the division of a "pie," which is discounted over time. Rubinstein showed that in the unique subgame perfect equilibrium of the perfect information game the first player's offer is accepted by the second player. More recent papers [1, 4, 7, 12-15, 26, 28, 31] have focussed on introducing incomplete information to the model. These papers have imposed various refinements and have then shown that the equilibrium involves delay,¹ which acts as a screening device.

This paper analyzes a bargaining game with imperfect, but complete, information and with simultaneous, rather than alternating, moves. The model is a twice repeated Nash [22] noncooperative bargaining game. Both players tender offers simultaneously, and if their offers agree then the pie is allocated accordingly. If no agreement is reached they try again,

* I thank Elchanan Ben-Porath, Joseph Farrell, David Kreps, and an associate editor for helpful conversations and comments. Financial support from the Miller Institute for Basic Research in Science and NSF Grant SES-8808133 is gratefully acknowledged.

¹ Gul, Sonnenschein, and Wilson [16] have shown that for a general class of these models (excluding Admati and Perry [1]) the delay disappears as the time between offers goes to zero.

whereupon if agreement is obtained the (discounted) pie is split, while if once again no agreement is attained the game ends without any division, or trade, occurring.

This model is obviously very specialized. It does, however, incorporate one feature of bargaining which the model of alternating moves fails to capture. This feature is that the bargaining parties may come to the table with predetermined expectations and offers, and will not be influenced (in the course of one meeting) by what the opponent says. This may be because incorporating the information revealed by the opponent takes more time than is spent in one meeting; for example, if the bargaining party must confer with its principal if deviations from its offer are to be accepted.²

A difficulty with the model of this paper is the extremely large set of subgame perfect (Selten [27]) equilibria. Even after the fairly strong notion of forwards induction (Kohlberg and Mertens [19]), is imposed, as captured in stability [19], the equilibrium set remains large. However, qualitative conclusions are obtained. The main objective in this paper is to examine the set of Pareto efficient outcomes which are supported as stable outcomes, and in particular how this set changes with the discount rate. The main result is that as the players become more patient, less Pareto efficient outcomes remain, until a point is reached beyond which only inefficient outcomes remain.³ Thus, patience causes delay.⁴ This also suggests that delegating bargaining to agents who must confer with the principal can cause inefficiencies.⁵ This necessity of inefficiency seems to me an important characteristic of noncooperative bargaining. In particular, this form of delay does not act as a device to screen among exogenously determined types, but as a method of indicating (endogenous) "stubbornness" (cf. [2, 30]).

This analysis is motivated above as clarifying certain aspects of bargaining situations. Another motivation is that this work contributes to the understanding of forwards induction and stability, and as such adds to recent papers [2, 14, 23, 30], which have examined the role of forwards induction and stability in selecting among equilibria in other games of imperfect

² Another feature of the simultaneous offer model is that it is *ex ante* symmetric, whereas there is a procedural asymmetry in the alternating move model.

³ In fact there exist subgame perfect equilibria which strongly Pareto dominate all the stable outcomes in a game with sufficiently patient players. This seems to raise doubts regarding the practice of focussing on Pareto dominating Nash equilibria (for example in the literature on renegotiation proof equilibrium [3, 9, 24, 29]).

⁴ This delay of course disappears as the time between the stages disappears, since only a two stage model is examined (cf. Gul, Sonnenschein, and Wilson [16]).

⁵ Delegation to agents who play a game *instead* of the original players is shown by Fershtman, Judd, and Kalai [11] to lead to cooperation.

information. Arguments based on forwards induction restrict beliefs about an opponent's "type" according to the possible gains to that "type" from deviating. Incomplete information bargaining models use such arguments [1, p. 349; 7, p. 202], where a player's "type" is a characteristic of his/her utility function (as in [17], and adverse selection models). Here the same intuition applies, where "type" refers to a strategy choice (as in moral hazard models). Thus, to the extent that the results here seem unreasonable, they may suggest that further work is needed to justify the use of forwards induction in other models (see also, [2]).

2. THE MODEL AND RESULTS

The (noncooperative) bargaining model of Nash [22] is specified by a utility possibility frontier, denoted by a non-increasing function $f: [0, 1] \rightarrow [0, 1]$, and a disagreement point $d = (0, 0)$. (The strategies and payoffs are determined in such a way that (as long as utilities are bounded) f and d can be normalized to have these values by taking positive affine transformations.) The (first period) strategies of players 1 and 2 are to specify utility allocations in $[0, 1]$ for player 1, denoted by s_1 and s_2 , respectively. If $s_1 \leq s_2$ then player 1 receives s_1 and 2 receives $f(s_2)$, while if $s_1 > s_2$ they both receive 0.

In this paper a two stage discrete version of this model is examined. Players are only allowed to suggest amounts in $S = \{m/n: m = 1, \dots, n - 1\}$, for some n . Furthermore, when $s_1 > s_2$ the players proceed to a second round. The second round strategies, denoted t_1 and t_2 respectively, are thus functions in $T = \{f: S \rightarrow S\}$. A fully specified strategy of a player is then a pair in $S \times T$. When the players choose strategies (s_1, t_1) and (s_2, t_2) , respectively, their payoffs are as follows. If in the first round $s_1 \leq s_2$, then (as in the one stage game) they receive s_1 and $f(s_2)$. Otherwise $s_2 > s_1$, and if $t_1(s_2) \leq t_2(s_1)$ (i.e., if 1's second period suggestion after observing s_2 in the first period is compatible with 2's second period suggestion after observing s_1 in the first period), they receive $\delta_1 t_1(s_2)$ and $\delta_2 t_2(s_1)$, respectively, where δ_i is player i 's discount rate. If their second period suggestions are not compatible (i.e., $t_1(s_2) > t_2(s_1)$), they both receive the disagreement allocation of 0. Henceforth discount rates in $\{\delta: s = s'/\delta \text{ for some } s, s' \text{ in } S\}$ are ruled out. This avoids (knife-edge) indifference between a first and a second period allocation, and thus serves the same purpose as tie breaking assumptions such as (A1) in [1] and (b-3) in [26]. The main result of this paper follows.

In order to state the main result of this paper we review the definition of a stable outcome. If a set of Nash equilibrium is stable [19, p. 1027] and each of these Nash equilibria induces the same probability distribution on

endpoints of the game, then this distribution is called a stable outcome [6, pp. 189–192].

PROPOSITION 1. *The set of Pareto efficient and stable outcomes is the set*

$$K = \{(s_1, f(s_1)) : s_1 > \delta_1 \text{ and } f(s_1) > \delta_2\}.$$

Furthermore, even if K is empty there exists a stable outcome.

Proof. The proof is an application of the forwards induction properties of stability [19, Proposition 6]. First it is demonstrated that any Pareto efficient outcome which is not in K is not stable. Consider the set of Nash equilibria, denoted by A , leading to any Pareto efficient outcome not in K , say the allocation $(s_1, f(s_1))$, where $s_1 \leq \delta_1$. (The case where $f(s_1) \leq \delta_2$ is similar.) For any $(s_2, t_2) \in A$ and for any s'_1 it is the case that $s_2 \leq \delta_1$, and $t_2(s'_1) < s_1/\delta$, since otherwise 1 would deviate to s'_1 . Therefore, for any $s'_1 > s_1$, a strategy for player 1 in $\{(s'_1, t_1) : t_1(s_2) < s_1/\delta_1\}$ is never a weak best reply against any strategy for player 2 which is in A . So consider the sub-(matrix)-game where these strategies are deleted. In this subgame, the strategies for player 2 in $\{(s_2, t_2) : t_2(s'_1) < s_1/\delta_1, s'_1 > s_1\}$ are weakly dominated. Now consider the further sub-(matrix)-game obtained by deleting these weakly dominated strategies. In this game none of the equilibria in A are Nash equilibria, since player 1 would prefer to deviate from s_1 to the next higher amount in S , to which 2 would respond with more than s_1/δ_1 . Theorem 6 in [19] implies that if A is a stable set, then there is an element in A which is an equilibrium in the sub-(matrix)-game which remains after such deletions. Thus A is not a stable set.

Next it is shown that any outcome in K is stable. In fact, these outcomes are hyperstable. In what follows let r_i denote strategies for the full game, namely pairs (s_i, t_i) , for player i . Let $(r_1, r_2) = ((s_1, t_1), (s_2, t_2))$ be an equilibrium which determines an outcome in K , and let A be the set of Nash equilibrium which yield that outcome. For any r'_2 such that (r_1, r_2) and (r_1, r'_2) yield the same payoffs for 2, (r_1, r'_2) is a Nash equilibrium. This is because the same payoffs can be obtained for 2 only if the players reach agreement in the first period. The payoffs for outcomes in K are greater than any possible second period payoff for either player. Thus player 1 cannot gain by deviating from r_1 . So A includes all strategy pairs that yield player 2 (respectively 1) the same payoff as (r_1, r_2) and are in a row (respectively column) of the game in which there is an outcome that is the same as the outcome given by (r_1, r_2) . Set ε to be less than the smallest difference among payoffs in the game, excluding equalities. If any of the payoffs in the normal form are then perturbed by an amount less than ε , one of the equilibria in A will still be a Nash equilibrium.

Finally, it is necessary to argue that a stable outcome exists when K is empty. Since this game has a non-generic extensive form, the claim does not follow from the generic arguments in Kohlberg and Mertens [19]. However, a completely mixed equilibrium can be constructed for this game, which is sufficient. This construction is straightforward. Q.E.D.

The characterization result in Proposition 1 has several nice properties. It demonstrates inefficiencies in the bargaining process which are a direct consequence of the players' patience, since for large enough δ_i , K is empty. On the other hand, as player's discount rate is increased, some efficient outcomes which are less favorable for that player become unstable. So, in a sense, each player benefits from increased patience—until a point beyond which they both lose. It is also noteworthy that the delay is not a signal of a player's preferences, but of his or her intent to play "tough." The result also shows how forwards induction rules out efficient allocations which are "extremely one sided."

Simultaneous move bargaining games have also been examined by Fershtman [10] and by Chatterjee and Samuelson [5]. The models differ from the above, and neither shows when delays must arise in equilibrium, as in Proposition 1. The former paper constructs an equilibrium which involves delay in a continuous time model, where it is assumed that players' demands must decrease over time. The latter paper considers an infinite horizon model, and shows (for several equilibrium concepts) that the set of equilibrium payoffs is not smaller than in the one period game. However, they do not examine the discrete game, which may lead to different results when forwards induction arguments are used (e.g., in the model above and in [2]).

In [25], if the discount rates are equal, then, as the player's impatience decreases, the equilibrium allocation converges to the Nash [21] cooperative bargaining solution. Thus, a noncooperative basis for the Nash solution is provided. We show next that in our model qualified support is obtained for a minor modification of the Kalai and Smorodinsky [18] solution (denoted MKS). When discount rates are equal, if any equilibrium outcome is Pareto efficient, then the MKS outcome is supported in an equilibrium (and no other Pareto efficient outcome has this property). That is, as the set of stable equilibrium outcomes that are Pareto optimal shrinks, it converges to the MKS outcome, before disappearing entirely. The KS solution to a normalized bargaining game with a convex utility possibility set is the allocation determined by the intersection of the diagonal with the Pareto frontier. Since this solution does not apply to the discrete version of the bargaining model (because this version is not convex) the solution must be modified slightly in order to retain the property that it is strongly Pareto optimal. The MKS solution is defined to

be any point on the Pareto frontier which weakly dominates the KS solution and is not itself weakly dominated.

COROLLARY 1. *Let $\delta_1 = \delta_2 = \delta$. Either the MKS solution is efficient and stable, or the efficient and stable set is empty. Furthermore, no other outcome has this property for all discount rates.*

Proof. Let (δ', δ') be the point on the diagonal which intersects the Pareto frontier, and let the MKS solution be (s_1, s_2) . For any $\delta < \delta'$, $s_1 > \delta$ and $s_2 > \delta$. Hence the MKS solution is in K . For δ sufficiently close to δ' , only the MKS solution satisfies the last two inequalities. Finally if $\delta > \delta'$, K is empty. Q.E.D.

An example is now provided to illustrate the problems which arise in generalizing the previous results to a model with more than two periods.⁶ The example shows that stability (in fact hyperstability) allows for efficient outcomes in a three-period model even when δ is large. It is worthwhile noting that the forwards induction used above may be weaker than that assumed in incomplete information bargaining models (cf. [1, p. 349]). Nevertheless in the present context it is not clear how the assumptions may be strengthened in order to extend the results to $n > 2$ periods. Despite the difficulties in attaining such an extension I believe that the intuition that delegating bargaining can cause delays by creating (signalling) incentives to play tough is of interest.

Consider a three-period game where in every period each player can either say low or high. Agreement is reached in the first period in which at least one player says low. If agreement is reached then low yields the payoff of 1, and high yields 2, while disagreement throughout the three periods yields zero. The discount rate is assumed to be δ for both players. This determines the payoff matrix in Fig. 1, where L; HL; HHL; HHH indicate low in the first period; high in the first and low in the second period; high in both the first and second periods and low in the third; and high in all three periods for player 1 (and lower-case letters are used for player 2).

It is easy to see that in the two-period version of this game (see Fig. 2) the outcome (2, 1) is not stable (hl is never a weak best reply against any strategy of player 1 which is used in a Nash equilibrium that yields (2, 1); HH is weakly dominated in the sub (matrix) game remaining after hl is deleted; l is weakly dominated after HH is deleted; and then HL is dominated, so (2, 1) is not an equilibrium outcome after these deletions, hence cannot be stable). Intuitively, by rejecting the outcome (2, 1) player 2 indicates that s/he will be demanding the high payoff in the second period. In the three-period model such a rejection is ambiguous—player 2 could be

⁶ The example actually involves a minor modification of the previous model.

	l	hl	hhl	hhh
L	1,1	1,2	1,2	1,2
HL	2,1	δ, δ	$\delta, 2\delta$	$\delta, 2\delta$
HHL	2,1	$2\delta, \delta$	δ^2, δ^2	$\delta^2, 2\delta^2$
HHH	2,1	$2\delta, \delta$	$2\delta^2, \delta^2$	0,0

FIGURE 1

indicating a demand for the high payoff in either the second or the third period. Formally it is proven below that (2, 1) is a hyperstable outcome of the game in Fig. 1.

PROPOSITION 2. *In the three-period model of Fig. 1 the outcome (2, 1) is hyperstable.*

Proof. First the set of Nash equilibria, denoted M , which yields (2, 1) is described:

$$M = \{ \langle (\alpha, HL, \beta, HHL; 1 - \alpha - \beta, HHH), 1 \rangle : 2\delta\alpha + \delta^2(1 - \alpha) \leq 1 \\ 2\delta\alpha + 2\delta^2\beta \leq 1, \alpha \in [0, 1] \beta \in [0, 1], \alpha + \beta \in [0, 1] \}.$$

Here $\langle (\alpha, HL; \beta, HHL; 1 - \alpha - \beta, HHH), 1 \rangle$ denotes the strategy pair consisting of player 1's playing HL (respectively HHL, HHH) with probability α (respectively $\beta, 1 - \alpha - \beta$) and player 2's playing l. In particular $\langle [(1 - \delta^2)/(2\delta - \delta^2), HL; (2\delta - 1)/(2\delta - \delta^2), HHH], 1 \rangle \in M$ and $\langle [(1 - \delta^2)/(2\delta - \delta^2), HL; (2\delta - 1)/4\delta - 2\delta^2), HHL; (2\delta - 1)/(4\delta - 2\delta^2), HHH], 1 \rangle \in M$. Denote the strategy of player 1 in the former equilibrium by q_1 and in the latter by r_1 .

Next it is argued that if any collection of payoffs in the matrix of Fig. 1 are perturbed by some collection of ϵ 's (where the ϵ 's are sufficiently small) then the perturbed game has a Nash equilibrium which is close to one of the equilibria in M . Only the essential parts of this argument are provided below.

Step 1. L and hl can be ignored since in no equilibrium close to M will either be assigned positive probability.

	l	hl	hh
L	1,1	1,2	1,2
HL	2,1	δ, δ	$\delta, 2\delta$
HH	2,1	$2\delta, \delta$	0,0

FIGURE 2

Step 2. Assume that the only perturbations are to player 1's payoffs in the cells corresponding to $\langle \text{HL}, 1 \rangle$, $\langle \text{HH}, 1 \rangle$ and $\langle \text{HHH}, 1 \rangle$, and denote these perturbations by ε_1 , ε_2 , and ε_3 , respectively. Perturbations of other payoffs in the matrix can be dealt with using arguments which involve the same steps and approximating equilibria as below—the arguments are similar but more tedious, hence are omitted.

Step 3. If $\varepsilon_3 \geq \varepsilon_1$ and $\varepsilon_3 \geq \varepsilon_2$ then $\langle \text{HHH}, 1 \rangle \in M$ is a Nash equilibrium of the perturbed game.

Step 4. If $\varepsilon_1 > \varepsilon_3$ and $\varepsilon_1 \geq \varepsilon_2$ then there is an equilibrium of the perturbed game close to $\langle q_1, 1 \rangle \in M$. In particular $\langle q_1, (1 - \mu, 1; \mu, \text{hhl}) \rangle$ is an equilibrium of the perturbed game, where μ solves

$$(2 + \varepsilon_1)(1 - \mu) + \delta\mu = (2 + \varepsilon_3)(1 - \mu) + 2\delta^2\mu \geq (2 + \varepsilon_2)(1 - \mu) + \delta^2\mu. \quad (1)$$

This equation makes player 1 indifferent between HL and HHH, with both preferred to HHL. Furthermore, as $(\varepsilon_1, \varepsilon_3) \rightarrow (0, 0)$, $\mu \rightarrow 0$, so this equilibrium is close to M .

Step 5. If $\varepsilon_2 > \varepsilon_3$ and $\varepsilon_2 > \varepsilon_1$ then either there is an equilibrium of the perturbed game close to $\langle q_1, 1 \rangle \in M$, or there is an equilibrium of the perturbed game close to $\langle r_1, 1 \rangle \in M$. The former holds if there is a $\mu \in [0, 1]$ such that (1) is satisfied. The latter is the case if there are $v, \eta \in [0, 1]$ with $v + \eta \in [0, 1]$ such that

$$\begin{aligned} (2 + \varepsilon_1)(1 - v - \eta) + \delta v + \delta\eta &= (2 + \varepsilon_3)(1 - v - \eta) + 2\delta^2 v \\ &= (2 + \varepsilon_2)(1 - v - \eta) + \delta^2 v + \delta^2 \eta. \end{aligned} \quad (2)$$

For μ solving Eq. (1), we can proceed as in Step 4. For v, η solving Eq. (2), if player 2 plays $r_2 = (1 - v - \eta, 1; v, \text{hhl}; \eta, \text{hhh})$ then player 1 is indifferent among HL, HHL, and HHL, so $\langle r_1, r_2 \rangle$ is an equilibrium of the perturbed game. It is easily verified that, as $(\varepsilon_1, \varepsilon_2, \varepsilon_3) \rightarrow (0, 0, 0)$, $v \rightarrow 0$ and $\eta \rightarrow 0$, and it can also be demonstrated, by comparing Eqs. (1) and (2), that at least one of these equations can be solved with μ, v, η , and $v + \eta$ in $[0, 1]$. (Q.E.D.)

REFERENCES

1. A. R. ADMATI AND M. PERRY, Strategic delay in bargaining, *Rev. Econ. Stud.* **54** (1987), 345–364.
2. E. BEN-PORATH AND E. DEKEL, "Coordination and the Potential for Self Sacrifice," Graduate School of Business Working Paper No. 984, Stanford University, 1988.
3. B. D. BERNHEIM AND D. RAY, Collective dynamic consistency in repeated games, mimeo, Department of Economics, Stanford University, 1987.
4. S. BIKHCHANDANI, "A Bargaining Model with Incomplete Information," Working Paper, Graduate School of Management, University of California at Los Angeles, 1985.

5. K. CHATTERJEE AND L. SAMUELSON, "Perfect Equilibrium Simultaneous-Offers Bargaining," Working Paper, University of Illinois, 1989.
6. I.-K. CHO AND D. M. KREPS, Signalling games and stable equilibria, *Quart. J. Econ.* **102** (1982), 179–221.
7. P. C. CRAMTON, Bargaining with incomplete information: An infinite horizon model with two-sided uncertainty, *Rev. Econ. Stud.* **51** (1984), 579–593.
8. Deleted in proof.
9. J. FARRELL AND E. MASKIN, "Renegotiation in Repeated Games," Working Paper 8759, Department of Economics, University of California at Berkeley, 1987.
10. C. FERSHTMAN, Simultaneous moves multi-person continuous time concession game, *Theory and Decision* **26** (1989), 81–90.
11. C. FERSHTMAN, K. JUDD, AND E. KALAI, "Cooperation through Delegation," CMSEMS DP No. 731, Northwestern University, 1987.
12. D. FUDENBERG, D. LEVINE, AND J. TIROLE, Infinite horizon models of bargaining with one-sided incomplete information, in "Bargaining with Incomplete Information" (A. Roth, Ed.), Cambridge University Press, Cambridge, UK, 1985.
13. D. FUDENBERG AND J. TIROLE, Sequential bargaining with incomplete information, *Rev. Econ. Stud.* **50** (1983), 221–247.
14. J. GLAZER AND A. WEISS, Collusive pricing, advertising and joint ventures: Strategically stable equilibria, mimeo, Bell Communications Research, 1987.
15. S. GROSSMAN AND M. PERRY, Sequential bargaining, under asymmetric information, *J. Econ. Theory* **39** (1985), 120–154.
16. F. GUL, H. SONNENSCHN, AND R. WILSON, Foundations of dynamic monopoly and the Coase conjecture, *J. Econ. Theory* **39** (1986), 1565–190.
17. J. C. HARSANYI, Games of incomplete information played by Bayesian players, *Management Sci.* **14** (1967–1968), 159–182, 320–333, 486–502.
18. E. KALAI AND M. SMORODINSKY, Other solutions to Nash's bargaining problem, *Econometrica* **43** (1975), 513–518.
19. E. KOHLBERG AND J.-F. MERTENS, On the strategic stability of equilibria, *Econometrica* **54** (1986), 1003–1037.
20. Deleted in proof.
21. J. NASH, The bargaining problem, *Econometrica* **18** (1950), 155–162.
22. J. NASH, Two-person cooperative games, *Econometrica* **21** (1953), 129–140.
23. M. J. OSBORNE, Signalling forward induction, and stability in finitely repeated games, mimeo, McMaster University, 1987.
24. D. PEARCE, Renegotiation-proof equilibria: Collective rationality and intertemporal cooperation, mimeo, Yale University, 1987.
25. A. RUBINSTEIN, Perfect equilibrium in a bargaining model, *Econometrica* **50** (1982), 97–110.
26. A. RUBINSTEIN, A bargaining model with incomplete information about preferences, *Econometrica* **50** (1985), 1151–1172.
27. R. SELTEN, Re-examination of the perfectness concept for equilibrium points in extensive games, *Int. J. Game Theory* **4** (1975), 15–55.
28. J. SOBEL AND I. TAKAHASHI, A multi-stage model of bargaining, *Rev. Econ. Stud.* **50** (1983), 411–426.
29. E. VAN DAMME, Renegotiation-proof equilibria in prisoner's dilemma, *J. Econ. Theory* **47** (1989), 206–217.
30. E. VAN DAMME, Stable equilibria and forward induction, *J. Econ. Theory* **48** (1989), 476–496.
31. R. B. WILSON, Game-theoretic analyses of trading processes, in "Advances in Economic Theory" (T. Bewley, Ed.), Proceedings of the Fifth World Congress, 1985.